UNCLASSIFIED

AD NUMBER
AD011220

CLASSIFICATION CHANGES

TO: unclassified

FROM: restricted

LIMITATION CHANGES

TO:

Approved for public release, distribution unlimited

FROM:

Distribution authorized to U.S. Gov't. agencies and their contractors; Foreign Government Information; JAN 1953. Other requests shall be referred to British Embassy, 3100 Massachusetts Avenue, NW, Washington, DC 20008.

AUTHORITY

DSTL, AVIA 28/3679, 14 May 2009; DSTL, AVIA 28/3679, 14 May 2009

THIS PAGE IS UNCLASSIFIED

MINISTRY OF SUPPLY

NATIONAL GAS TURBINE ESTABLISHMENT
PYESTOCK, HANTS.   WHETSTONE, LEICS.

REPORT No. R.132

PRESSURE PULSES CAUSED BY A
CHANGE IN FUEL AIR RATIO IN A
SIMPLY STABILISED BURNING ZONE

by

D.G. STEWART and M.V. NESBITT

JANUARY, 1953

NATIONAL GAS TURBINE ESTABLISHMENT

Pressure Pulses Caused by a Change in Fuel Air Ratio

in a Simply Stabilised Burning Zone.

- by -

D. G. Stewart and K. V. Nesbitt

SUMMARY

This report describes an experimental investigation of the pressure transient caused by a sudden change in fuel-air ratio in a simply stabilised combustion zone such as that of a ramjet or reheat system. This information is required for a full investigation of combustion excited oscillations in such systems.

The experimental results, which show considerable scatter owing to difficulties of measurement and random effects in the transient process, are compared with theoretical results and, when allowance for friction is made, these are in good agreement for small transients, the divergence between theory and experiment for large transients possibly being caused by simplifications made in the theoretical calculations.

It is considered that the data finally obtained should apply reasonably well to transient combustion processes in other chambers using other fuels, provided stoichiometric fuel-air ratio is not exceeded.

<u>CONTENTS</u>

<u>TABLES</u>

<u>APPENDICES</u>

<u>ILLUSTRATIONS</u>

ILLUSTRATIONS (cont'd.)

1.0   Introduction

The present work is part of a general investigation of combustion
excited oscillations in ramjet and reheat chambers which commenced with the
work of Ref. 1. A detailed investigation of the mechanism of oscillation
involved and the factors affecting it is reported in Ref. 2 but the follow-
ing brief explanation of the mechanism studied there will be given here:-

Referring to Fig. 1, if a small random pressure disturbance arises
somewhere in the combustion region at B, it causes pressure waves to
propagate up and downstream. The upstream wave causes a reduction in air
mass flow and so, as the fuel flow is sensibly constant, a quantity of
richer mixture forms as the upstream pressure wave passes the fuel injector.
This rich mixture, on flowing down to the combustion zone, releases extra
energy, and so gives rise to a second pressure disturbance. In this way a
regular succession of pressure pulses may occur. This simple explanation
does not fully describe the complex oscillations dealt with in Ref. 2 in
which secondary wave reflections are important, but it is sufficient to
provide a background for the present work.

A full investigation of instability of this type would require the
cycle of oscillation to be amenable to calculation, thus allowing theoretical
predictions of stability limits and amplitudes of oscillation in any chamber.
Although this ideal cannot be fully achieved owing to complicated secondary
effects, it is still desirable to approach it as far as possible and to know
quantitatively what occurs during a cycle.

The greatest uncertainty in analysing the cycle of oscillation is the
effect of a change in fuel-air ratio in producing a pressure transient. A
quantitative knowledge of this effect should be useful, not only with
reference to the mechanism of instability mentioned, but also in connection
with other phenomena. For example, in a ramjet operating with an unstable
intake, the amplification by the combustion zone of flow disturbances arising
in the intake could be calculated. It should be noted that, since the com-
bustion region is, in these cases, some distance from the fuel injector, it
is immaterial, as far as the combustion behaviour is concerned, whether a
change in fuel-air ratio is brought about by a change in fuel or in air mass
flow.

In this report an experimental study of the transient effects of a
change in fuel-air ratio is described and the results of this work are
compared with various theoretical treatments of the same problem.

2.0   Theory

2.1   General

The problem to be considered is that of a homogeneous mixture burning
behind a flame stabilising baffle in a uniform duct. If steady conditions
are disturbed by suddenly altering the fuel-air ratio it is required to
find the resultant pressure and flow disturbances. The nature of the change
in fuel-air ratio may be a pulse (i.e. an increase or decrease followed
by a return to the initial value), or, in the simplest case, a transient
(i.e. a unidirectional change followed by a new constant value).

The qualitative flow behaviour may be illustrated by reference to
Fig. 2. Fig. 2A shows the distance-time (or x - t) diagram for steady
conditions in which the combustible mixture flows towards a flame stabiliser
A where it ignites and burns, combustion being assumed complete at the

section B. Lines such as CD represent the path of successive gas layers down the tube as they are accelerated through the burning region.

Consider now the case of a transient increase in fuel-air ratio in such a system (Fig. 2B). Again A and B represent the flame stabilisers and original completion of combustion. Line EF represents the path of the gas layer in which the fuel-air ratio starts to increase and it is assumed to increase up to the layer whose path is GH, after which the fuel-air ratio remains constant at the higher value. The line of completion of combustion, which is now not a fixed point in the duct, becomes BJKL.

Theoretical treatments of this process and full theoretical discussions are obtained in Ref. 3, 4 and 5, some of the results of which are referred to later and compared with the present experimental results. The purpose of this section is to draw attention to the theory as an aid to interpreting the experimental results. In particular it is necessary to find from the theory some indication as to how the various experimental variables are likely to enter into the final result and how they may best be combined in finding a final correlation of experimental results.

An exact treatment of the problem can be made only if the combustion behaviour is known throughout the process and it is here that the greatest difficulty arises. The combustion rate during steady combustion may be determined experimentally and this has been done on the present test rig. (See Appendix I). However, although these results will apply to the steady state before the transient, they give no indication of the combustion rate during or immediately after the transient and it would be possible to find this rate only by means of very elaborate instrumentation. There appears to be no theoretical way of determining precisely the combustion rate during a transient since this is likely to depend on both pressure and turbulence effects. It may be expected, for example, that the sudden onset of unsteady flow during the transient can so increase the turbulence in the combustion zone that the combustion rate may rise during the transient and it is also possible that if the initial steady combustion is inefficient, the efficiency may rise, thus effectively increasing the magnitude of the fuel transient.

Because of these uncertainties it is impossible at the moment to calculate exactly what occurs during a transient and the best that can be done is to calculate on the basis of definite assumptions and to assume that the results so calculated will indicate the trend of events although they may not be expected to be in perfect numerical agreement with the experimental results. The complex analyses are very greatly simplified if the combustion behaviour is assumed such that before, during and after the transient, the time rate of entropy increase of a burning gas layer remains constant (or at least that this rate depends only on duct position) and this assumption is made in the theoretical calculations below.

It should be noted that the comprehensive theory of Ref. 6 allows calculations to be made for any law of burning without this limitation on entropy rate but as the actual burning law is not yet known, calculations to compare with experiment have been made based only on the simple assumption. Various theoretical calculations are available and the following will be considered, in order of increasing complexity:-

1.   Small pulse calculation, valid only for infinitesimal combustion transients, (See Ref. 3)

(a)   considering zero initial fuel flow,

(b)   considering a large initial fuel flow in which case
the pressure pulses are modified by attenuation and
reflection in the large burning zone.

2.   Mathematical large pulse calculation consisting of an integra-
tion of the small pulse theory, (See Ref. 3)

(a)   neglecting all attenuations and reflections and assuming,
fictitiously, that successive increments of the pressure
pulses are measured where they are originally produced,

(b)   considering the first approximation to the attenuation
of the pressure pulses in the burning and burnt zones
but ignoring reflections.

3.   Complete graphical calculation for a large pulse and large
initial fuel flow, all attenuations and reflections being considered, the
result applying only for one typical magnitude of combustion transient.
(See Ref. 3).

2.2   Small pulse calculation

It is shown in Ref 3 that, for a small fuel transient, zero initial
fuel flow and a constant rate of entropy increase through the burning zone,
the pressure transient is given by:

$$\frac{1 \pm M_0}{M_0} \quad \frac{\Delta p}{p} = \frac{\gamma}{2 \, C_p} \quad \Delta S \qquad \dots\dots\dots\dots\dots\dots\dots(1)$$

where M_0 = initial steady Mach number

p = "   "   pressure

Δp = transient increase in pressure

ΔS = "   "   "   entropy due to combustion
                                of additional fuel

C_p = specific heat at constant pressure

γ = specific heat ratio

and the $\pm$ sign refers to downstream and upstream pulses respectively.

Using hydrogen as fuel it is shown in Appendix II that equation (1)
becomes:

$$\frac{1 \pm M_0}{M_0} \quad \frac{\Delta p}{p} = 0.267 \quad \Delta(F/_A) \qquad \dots\dots\dots\dots\dots\dots(2)$$

where $\Delta(F/_A)$ = transient increase in fuel-air ratio.

The important conclusions from this theory for small pulses and zero
initial fuel flow are:-

(1)   The magnitudes of pressure transient and fuel transient are

directly related indeendent of time effects. equation (2) giving the slo; at the origin of the urve of pressure transient against fuel transient.

(2)   The effect of initial Mach number is simply expressed by factor

$$\frac{1 \pm M_0}{M_0}$$

For a large inital fuel flow, pulse reflections and attenuations be dealt with by the method of Ref. (5) and this has been done in Appe II for an initial hydrogen flow of 20 per cent stoichiometric burning w 100 per cent efficiency with initial cold Mach numbers of 0.10 and 0.20. The results may be expressed as:-

$$\frac{1 \pm M_0}{M_0} \frac{\Delta p}{p} = k \; \Delta(F/A) \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots ($$

where the factor $\dfrac{1 \pm}{M_0}$ is included for reference to the original formul but does not indicate that the effect of the initial cold Mach number is so expressed. Values of k (from equations (7) and (8) of Appendix II) are as follows:-

	Upstream pulse	Downstream pulse
Zero initial fuel flow,		
any Mach number	0.26 ?	0.26 ?
Initial F/A 20 per cent		
stoichiometric		
$M_0 = 0.1$	0.111	0.178
$M_0 = 0.2$	0.119	0.189

These numerical values are compared with experimental results in Section 4 and Figs. 8 and 9. They indicate that, owing to attenuations and reflections in the burning zone, the simple formula for zero initial fuel flow does not apply for not initial conditions although the effect of initial Mach number is still quite closely expressed by the factor $\dfrac{1 \pm M_0}{M_0}$

2.3   Mathematical large pulse calculation

The derivation of large pulse theory from that for a small pulse is explained in Appendix II and only the results of calculations described in that Appendix will be discussed here.

In the Appendix calculations are made of the pressure transients for fuel transients up to $\frac{F}{A} = 14.0 \times 10^{-3}$ assuming zero initial fuel flow and initial Mach number = 0.20.

Calculations were made on two bases:-

(a)   Neglecting all attenuations and reflections of pressure pulses in the burning and burnt regions.

(b)   Considering the first approximation to attenuation.

Fuller discussion of what is meant by these two assumptions is contained in Appendix II.

The results of these calculations are shown in Figs. 8 and 9 and they are discussed in Section 4.2. In particular the following should be noted:-

(1)   The slope at the origin of the large pulse result is as given by the small pulse calculation.

(2)   The calculations made do not specifically indicate effects of initial fuel flow and Mach number on the transient, although it may be expected that the same trends will apply as for the small pulse calculation.

2.4   Complete graphical calculation

The calculation of Ref. 4, apart from the limitation of constant entropy rate, is accurate in considering all attenuations and reflections without approximations.

The calculation of Ref. 4 applies to an initial cold Mach number of 0.20 and for one fixed set of data which, using the appropriate initial conditions, leads as shown in Appendix II to:-

Initial fuel-air ratio before transient   = 2.98×10^{-3}

Increase in fuel-air ratio during transient = 6.45×10^{-3}

The results of the claculation are given in Ref. 4 and show the time variation of upstream and downstream pressure and velocity during the transient. The results of interest are the peak pressures which are:-

For the upstream pulse $\frac{\Delta p}{p}$ = 0.300

For the downstream pulse $\frac{\Delta p}{p}$ = 0.345 (Case 1)

$= 0.255$ (Case 2)

where p = initial steady pressure before transient

Δp = maximum pressure increase during transient.

Here case 1 refers to the downstream pressure which would be measured at a point where the peak pressure coincided with peak temperature (i.e. at a point corresponding to the longest flame lengths, K, Fig 2B). Case 2 refers to the downstream peak pressure when the temperature at the point of pressure measurement is still at the lower value(i.e. measuring point well downstream of burning region so that the pressure wave reaches the measuring point before the region of increased temperature reaches it.)

The significant conclusions from this work, apart from the actual numerical values which are compared with experimental results and the other

theories in Section 4.2 and Figs. 8 and 9 are:-

(1) The magnitude of the downstream pulse will depend in complex fashion on the initial Mach number and fuel flow and on the point of pressure measurements because of the existence of the two cases mentioned depending on whether or not the high pressure and high temperature regions coincide. The upstream pulse will not be thus dependent.

(2) It is shown in Appendix II that, for a given initial Mach number, one important parameter specifying the problem is the ratio of the time for peak pressure to occur in the pulse to the time of change in fuel-air ratio. Although it is impossible to decide what is the dependence on this ratio it is conceivable that fuel-air ratio and pressure transient magnitudes and times may be related such that the rate of increase of pressure depends only on the rate of increase of fuel-air ratio.

3.0 Experiment

3.1 Rig description and experimental technique

These experiments were carried out with a duct which simulated conditions in a ramjet. The duct was of 3 inches diameter stainless steel with a pitot intake and diffuser at one end blown by a free jet and having separated fuel injectors and flame stabilisers. No attempt was made at actually scaling a ramjet, but rather the wave conditions appertaining to a ramjet were treated on a smaller scale. A schematic diagram of the rig set up is given in Fig. 3.

The fuel used was hydrogen. A bank of six cylinders pressurised to 3,500 p.s.i. supplied fuel to the rig through two pressure reducing valves and a valve network which enabled the rig to be run at a steady fuel flow while at the same time a rapid pulse of fuel could be supplied. This valve network is illustrated in Fig. 3. The pressure reducing valves were always set to give a pressure of about 400 p.s.i. immediately after them, any fluctuations through them being smoothed by the damping cylinder. After passing through the main fuel control valve the fuel entered a bifurcated section, each branch of which contained a throttle valve and one of which also contained a quick operating solenoid valve. The two branches joined a common tube running to a fuel distributor which was designed to have the minimum volume commensurate with distribution to seven fuel spoke supply lines. The fuel spoke supply lines were of equal length so that any transient phenomenon reaching the fuel distributor would be transmitted to all the fuel spokes in the same time interval. Using this arrangement a large increase in fuel flow could be applied in less than 5 milliseconds.

The rate of fuel supply was read from a pressure gauge tapped into the fuel distributor. This gauge was calibrated for hydrogen flow against a small British Standard orifice plate over a range of back pressures. It was found that for the range encountered under running conditions, back pressure had no significant effect on fuel flow as the injection holes were running choked. The seven spoke fuel injector was designed to give as uniform as possible fuel distribution over the duct.

The setting of the fuel system to supply any desired steady fuel flow while at the same time being able to supply any desired fuel transient was accomplished by manipulating throttle valve A and the main fuel control valve until the flow gauge gave the desired reading and the pressure in

the bifurcated section was such as to give the desired transient. The fuel
transient was provided by the sudden opening of the solenoid valve which
allows a rapid rate of pressure rise at the distributor. Throttle valve
B was used to control the rate of rise of pressure so that the initial slope
of the transient could be varied at will.

The pressure variations in the duct were measured by three capacity
type water cooled pressure pick-ups mounted at the positions shown in Fig. 3.
The outputs from these pick-ups were fed into three channels of a four channel,
amplitude modulated electronic recorder. The fourth channel recieved signals
from a pick-up in one of the fuel spoke feed lines so that it recorded
transient fuel phenomena. The movements of the four cathode ray oscillos-
cope spots, together with a 50 cycle time spot, were recorded by a 70 m.m.
Avimo strip camera operating at a paper speed of 36 inches per second.

The experimental procedure was as follows:-

With the airflow set at any convenient value the fuel flow was set
to give the desired steady flow while at the same time the fuel pressure
in the branched section was adjusted to give the required fuel transient.
Following this operation the Mach number was set to a specified nominal
value using a graph connecting Mach number with static and dynamic duct
pressures. Air flow was measured by means of a pitot comb connected to a
bank of manometers and it was adjusted by manipulation of the main air
control valve.

Having set the fuel and air flow, readings were taken of air tempera-
ture, air-flow manometers, fuel temperature and pressures and exhaust tempera-
ture using a ten point traverse. The camera was started and, after giving
it approximately half a second to speed up, the solenoid valve was opened.

Care was needed in the handling of the pressure pick-ups and to
obviate errors the pick-ups were statically calibrated in situ before and
after each run by sealing the ends of the duct and pressurising.

3.2   Results and analysing technique

Typical records of duct conditions occurring when a fuel transient
was applied are shown in Fig 4. The four channels represent the following:-

Channel 1 - duct pressure upstream of injector and stabiliser.

Channel 2 - duct pressure between injector and stabiliser.

Channel 3 - duct pressure downstream of stabiliser.

Channel 4 - fuel pressure.

Figs. 4A and 4B show typical records for small and large transients
respectively. Fig 4C shows the record obtained when the solenoid valve
was closed after a transient before the camera was stopped. Although it
was not primarily intended to investigate this "rarefaction transient",
the few records obtained in this way have been analysed and compared with
the "compression Transients".

Correlation of results was attempted on many lines (see Section 4.1)
but only two types of measurement from the records were required. One of
these, the least satisfactory, required finally the rate of pressure rise

and the rate of fuel flow rise to be determined, which involved measurement
of the gradients of the traces as the transient occurred. Measurements of
the gradient of the fuel transient were quite straightforward and reproduci-
bility was good but measurements of the gradients of the duct pressure
transients were exceedingly difficult. The records show that under steady
conditions there is quite a deal of pressure disturbance due to duct turbu-
lence possibly magnified by combustion and during a transient this steady
state "hash" tends to distort the trace making it difficult to determine
the true mean path of pressure rise. For this reason, when the gradient
was measured it was possible to make an error of up to ± 2 degrees in measur-
ing the angle, and as the angle of rise was usually greater than 70 degrees,
this error was magnified considerably when the tangent of the angle was
used in calculations.

Other methods of calculation required the measurement of the amplitude
of fuel flow and duct pressure changes. As before, the measurement of the
change in fuel flow was an easy matter, but in the measurement of amplitude
of pressure change the "hash" caused some uncertainty as to what was the true
peak of the transient. However, this method had the advantage over the
gradient method that errors made were not amplified in the calculations.

Several correlations, including the gradient type, required a measure-
ment of the time intervals over which the fuel flow and pressure rose. In
all cases it was difficult to decide exactly when the rise began and ended,
and in all correlations involving time intervals the results were suspect
for this reason.

3.3   Notes on reproducibility and accuracy

The accuracy of readings obtained in these experiments was not high
for several reasons. In all cases there was a certain amount of pressure
fluctuation due to normal turbulence in the duct and to the resulting
unsteady combustion. This distorted the trace on the film so that the true
path of the pressure change produced by a fuel transient was not always
clear. Where measurement of the rate of pressure change was required, i.e.
measurement of the gradient of the change as shown on the trace, this back-
ground made it quite difficult to decide what was the true gradient and as
mentioned before errors made in the measurement of this angle were considerably
magnified when its tangent was used in calculations of the rate of change
of pressure.

When measurements were made of the amplitude of the pressure change,
the background again made it difficult to determine the true peak pressure
but as there was no magnification of error, as was the case with gradient
measurements, this method gave a more accurate result.

Another source of error was the lack of control over the thickness
of the trace appearing on the film. The maximum deflection measured was
of the order of 0.3 inch and generally speaking the measured deflections
were about 0.15 inch so that the trace, if too thick, could be a source
of error. The thickness of the trace depended upon the exposure given to
the film by the cathode tube spot and hence upon the rate of change of
pressure which meant that measurements of small deflections were liable
to larger errors due to this cause than larger deflections. This does
not necessarily mean that the larger transients were measured more
accurately, since often to record large transients the gain of the
amplifiers needed to be lowered, resulting in a smaller deflection of the

cathode tube spot.

With gradient measurements thick traces led to further uncertainty as to what was the true gradient of the change. As before large deflections giving thinner traces led to more accurate results.

In addition to all these measurement errors there is the ever-present effect of non-reproducibility of the transient process due to uncontrollable variation of turbulence and other factors. This effect is likely to be far more serious than that encountered with steady combustion processes.

In an attempt to get some quantitative ideas of the magnitude of errors occurring, a run was carried out in which twenty-eight readings were taken at the same rig setting. The results were computed to determine rate of pressure rise and amplitude of pressure rise and the results are shown in Table 1. As an indication of the reproducibility obtainable the range of values obtained from each channel during this run are shown.

TABLE I

Accuracy of experimental results

Quantity	Rate of pressure rise $\frac{\Delta P}{\Delta t}$	Pressure Amplitude ΔP
Mean fuel value Standard deviation of a single observation Standard deviation of arithmetic mean Range of values obtained	$\frac{\Delta P}{\Delta t}$ = 3.772 milli.lb./sec. 0.10985 0.55% 3.605 to 4.05 milli.lb./sec.	ΔP = 5.72 milli.lb./sec. There was no measurable error in the measurement of the amplitude of the fuel transient
Channel 1. Mean value Standard deviation of a single observation Standard deviation of arithmetic mean Range of values obtained	$\Delta P / _{\Delta t}$ = 1697 lb./in.^{2}/sec. 421 lb./in.^{2}/sec. 4.69% 1100 to 2660 lb./in.^{2}/sec.	ΔP_1 = 4.12 in. Hg. 0.276 in. Hg. 1.27 % 3.51 to 4.86 in. Hg.
Channel 2. Mean value Standard deviation of a single observation Standard deviation of arithmetic mean Range of values obtained	$\Delta P / _{\Delta t}$ = 1546 lb./in.^{2}/sec. 264 lb./in.^{2}/sec. 3.22% 1175 to 2010 lb./in.^{2}/sec.	ΔP_2 = 4.84 in. Hg. 0.282 in. Hg. 1.1% 4.30 to 5.37 in. Hg.
Channel 3. Mean value Standard deviation of a single observation Standard deviation of arithmetic mean Range of values obtained	$\Delta P / _{\Delta t}$ = 2369 lb./in.^{2}/sec. 832 lb./in.^{2}/sec. 6.75 % 1065 to 4400 lb./in.^{2}/sec.	ΔP_3 = 5.75 in. Hg. 0.974 in. Hg. 3.2 % 4.33 to 8.65 in. Hg.

The two methods of obtaining a final result can be weighed against each other by comparison of the standard deviations of the arithmetic means for each channel (see Table 1). Such a comparison shows that amplitude measurements give errors of a third to a half of errors incurred in the other method. Examination of the standard deviations of single observations indicates that for the determination of rate of pressure rise these errors are of the order of 17 - 35 per cent of the value read while for the pressure amplitude measurements they are of the order of 6 - 17 per cent depending upon which channel is considered. These results indicate that for single observations measurement of rate of pressure rise is likely to be in far greater error than measurement of pressure amplitude.

The greater part of the error in the final results arises from transient pressure measurement and on this basis the standard deviation of a single observation is approximately 6 per cent for channels 1 and 2 and 17 per cent for channel 3.

4.0   Results and discussion

4.1   Correlation of experimental results

The experimental investigation contains many variables such as initial Mach number and fuel flow, magnitude of transient and rate of increase of fuel flow and pressure during the transient. It is the purpose of this section to establish how the experimental results can best be expressed to take full account of all experimental variables.

The theoretical treatments form a guide to the way in which the results should be expressed but they do not give complete guidance as, in some respects, the various treatments lead to different conclusions. The important conclusions reached in Section 2 are as follows:-

Small pulse calculation

(1)   For a small transient and zero initial fuel, this predicts that actual transient peak magnitudes should correlate, independent of time effects.

(2)   For a small transient and zero initial fuel, it predicts that the effect of initial Mach number is simply that the pressure pulse magnitude is proportional to $\dfrac{M_o}{1 \pm M_o}$

(3)   For a small transient and inital fuel flow other than zero, the effect of initial Mach number may still be expressed approximately by the factor $\dfrac{M_o}{1 \pm M_o}$ and the effect of increased initial fuel flow is to decrease the "reduced" pressure transient $\dfrac{1 \pm M_o}{M_o} \dfrac{\Delta p}{p}$

Mathematical large pulse calculation

(4)   The results of this calculation do not indicate how initial Mach number and fuel flow should enter the correlation but the effect might be expected to be similar to that of small pulse theory.

Complete graphical calculation

(5)   This makes no prediction of the general effect of initial Mach number and fuel-air ratio, but suggests that both these factors will be

particularly important for the downstream pulse due to the effect of coincidence of peak pressure and peak temperature.

(6)   It suggests that rate of increase of pressure should depend on rate of increase of fuel-air ratio rather than that the peak values of pressure and fuel transient should be related.

In order to obtain an experimental relation the results have been processed and correlations attempted in many ways bearing in mind the theoretical predictions and other considerations to be discussed. The difficulty of deciding the best agreement is very greatly increased by the large inherent experimental scatter which usually overshadows any trend being investigated and which consequently makes it difficult to decide which is the best of the various attempts.

The attempted correlations differ in the following respects:-

They may be of:-

      (1)   Rates against rates
      (2)   Rates against magnitudes
      (3)   Magnitudes against magnitudes.

Rates may be determined:

      (1)   By measurements of slopes
      (2)   By measurements of finite increments.

The fuel parameter may be expressed as:

      (1)   Proportional change in fuel flow
      (2)   Change in fuel-air ratio.

In addition various assumptions may be made regarding the effect of the additional fuel and of the initial combustion efficiency.

The correlations attempted are summarised in Table 2 and rather than show all results, which ranged from good to very bad, the conclusions from each attempt are shown in the Table. All pick up points and Mach numbers have not been used for every attempt. The various correlations will now be discussed,

Correlation No. 1 (Table 2)

At first the only theoretical results available were those of Ref. 4 and, in view of the conclusions (5) and (6) above, correlation of the experimental results was tried by plotting rate of increase of pressure for all three pick up points against rate of increase of fuel-air ratio, both these rates being determined from the slope of the pressure traces at the point of initial rise. In view of conclusion(5) separate plots were made for each initial Mach number of 0.15, 0.20, 0.25 and 0.30 and consideration was also given to the effect of initial fuel flow.

This yielded correlations for the various pick up points and Mach numbers ranging from fairly good to bad, the average being fair. The scatter was great but nevertheless the trend of the effect of initial Mach number was evident. There was however nothing to suggest that initial fuel flow was of any consequence and it was concluded that this was an unimportant variable.

This method of correlation suffered from great inaccuracies in computing rates from tangents as discussed in Section 3.2 and was finally abandoned for this reason.

It should be noted that the few results obtained with 7 in. and 49 in. lengths between injector and stabiliser in place of the standard 25 in. length yielded average results of very different magnitudes and it was realised that this is due to diffusion altering the distribution of fuel in the transient before it reaches the stabiliser. This effect will thus become important when rate of increase of pressure is compared with rate of increase of fuel-air ratio and will make the results dependent, not only on the injector-stabiliser length but also on the duct diameter and degree of turbulence. If results were obtained in this way, they would thus be of only limited application.

Correlations Nos. 2 - 5 (Table 2)

As an alternative, rates of increase of pressure and fuel flow were obtained by measuring the increments in pressure and fuel flow and dividing by a measured time increment. The time increment used was the mean of the time to reach peak pressure as recorded by pick ups 2 and 3, as pick up 1, being well upstream, suffered from a wave steepening effect and did not record such consistent time intervals as the other two. This mean time interval was used in calculating both pressure and fuel rates, so eliminating the diffusion effects referred to in Correlation 1.

Several correlations were attempted since it was uncertain whether the fuel variable could best be expressed by a proportional change in fuel flow $\frac{\Delta F}{F_1}$ or by a change in fuel-air ratio $\Delta (F/_A)$. (For a given Mach number A is fairly constant and the latter becomes closely proportional to ΔF.) In addition it was thought that when a fuel increment ΔF is introduced to a steadily burning region with initial fuel flow F_1, the transient may so disturb the region that the initial fuel F_1 may burn more rapidly than previously, so that near the start of the combustion zone the additional fuel burnt may actually be $(F_1 + \Delta F)$ rather than ΔF. For this reason correlations were attempted using both ΔF and $F_1 + \Delta F$ as a measure of the fuel increment.

As before, separate plots were made for each Mach number and each initial fuel flow and it was again concluded that whilst the Mach number effect was definite, that of initial fuel flow was masked by scatter. The correlations obtained were better than in No. 1 but still showed considerable scatter. The most important conclusion to be drawn was that correlation was far better in terms of fuel air ratio than in terms of proportional fuel flow. (See Table 2).

Correlations Nos. 6 - 8 (Table 2)

Following on the previous ideas regarding the effect of initial fuel flow it was thought that if the initial steady state is inefficient and the transient causes an efficiency increase to unity, the fuel increment may best be expressed by $\Delta F + (1 - \eta) F_1$ rather than by ΔF or by $\Delta F + F_1$. Also in considering the proportional increase in heat release the denominator should be ηF_1 rather than F_1.

Attempts along these lines showed no improvement in the quality of correlation obtained and it was concluded that the computing complexity involved in processing the results thus was not justified.

Correlations Nos. 9 - 12 (Table 2)

An attempt was made to correlate rates with magnitudes and vice versa, expressing fuel change in terms of fuel-air ratio since that had previously shown best results. Both $\Delta(F/_A)$ and $\dfrac{\Delta F + F_1}{A}$ were used to express the fuel increment.

The results of these correlations were bad and these attempts were abandoned.

Correlations Nos. 13 - 17 (Table 2)

About this time the results of Ref. 3 became available and in view of the prediction that magnitudes rather than rates are significant, correlation was attempted along these lines. Results were no better than those obtained previously but were, in general, no worse and the simplification of neglecting time effects was attractive. The results confirmed previous findings that fuel transient is better expressed in terms of fuel air ratio than in terms of proportional fuel flow.

A further prediction of Ref. 3, that Mach number variation can be expressed by the factor $\dfrac{1 \pm M_0}{M_0}$ was taken into account in correlations 16 and 17 and it was found that reasonable correlation of all Mach numbers was obtained in this way on the one graph. These results are considered to be the best and most useful obtained and are reproduced in Figs. 5, 6 and 7. It should be noted that the scatter for the downstream pulse (pick up 3) is worse than that of the other two as predicted in Section 2.2, item 2.

In correlations 16 and 17 the fuel increment has been expressed in two forms, $\Delta(F/_A)$ and $\dfrac{\Delta F + F_1}{A}$ representing the two extreme assumptions of the effects on the initial fuel of the fuel transient. It is difficult to choose which gives the better result but that in terms of $\Delta(F/_A)$ is probably better and is certainly the simplest and easiest to make use of.

It should be noted that the factor $\dfrac{1 \pm M}{M}$, based on Mach number at the point of pulse production, occurs in all theoretical treatments, including the general one of Ref. 6, but the factor $\dfrac{1 \pm M_0}{M_0}$, based on initial Mach number, is predicted only for a small transient and for zero initial fuel flow. Nevertheless its use for the experimental results of Figs. 5, 6 and 7 yields satisfactory calculations.

Included also in Figs. 5 and 6 are the results of the few rarefaction transients obtained, as described in section 3.2 when the fuel flow was suddenly decreased. The values of $\dfrac{\Delta F}{A}$ and $\dfrac{\Delta F}{F}$ shown are actually negative and indicate the same trend as for positive transients so that it may be concluded that the same mean line applies for compression or rarefaction pulses. Correlation of the rarefaction pulses on the basis of $\dfrac{\Delta F + F}{A}$ has not been attempted as this would be meaningless.

TABLE 2 - ATTEMPTED EXPERIMENTAL CORRELATIONS

CORRELATION OBTAINED

G = Good	FB = Fairly Bad
FG = Fairly Good	B = Bad
F = Fair	VB = Very Bad

I = Insufficient Data

Column legends:

- CORRELATION NUMBER
- BASIS: R = Rate against Rate; RM = Rate against Magnitude; M = Magnitude against Magnitude
- RATE MEASUREMENT: T = By Tangents; I = By Increments
- FUEL PARAMETER: P = Proportional change; P' = % Change in fuel-air ratio
- COMBUSTION ASSUMPTIONS: A = Additional fuel causes transient; I = Initial and additional fuel combine to form transient; E = Combustion efficiency considered
- DEPENDENT VARIABLE: n = P.U. number
- INDEPENDENT VARIABLE: av = Mean of P.U. 2 and 3

CORRELATION NUMBER	BASIS	RATE MEASUREMENT	FUEL PARAMETER	COMBUSTION ASSUMPTIONS	DEPENDENT VARIABLE	INDEPENDENT VARIABLE	M = 0.15 Pick Up No. 1	M = 0.15 Pick Up No. 2	M = 0.15 Pick Up No. 3	M = 0.20 Pick Up No. 1	M = 0.20 Pick Up No. 2	M = 0.20 Pick Up No. 3	M = 0.25 Pick Up No. 1	M = 0.25 Pick Up No. 2	M = 0.25 Pick Up No. 3	M = 0.30 Pick Up No. 1	M = 0.30 Pick Up No. 2	M = 0.30 Pick Up No. 3	CONCLUSIONS
1	R	T	P	A	$\left(\frac{dp}{dt}\right)_n$	$\frac{d(F/A)}{dt}$	B	B	F	G	B	FB	B	F	FG	BI	F	FG	Fair correlation. Insufficient accuracy of measurement.
2	R	I	P	A	$\frac{\Delta p_n}{\Delta t_{av}}$	$\frac{\Delta(F/A)}{\Delta t_{av}}$	FG	G		F	F	FB							General conclusions are that better correlations are obtained in terms of fuel-air ratio than in terms of proportional fuel flow.
3	R	I	P	I	$\frac{\Delta p_n}{\Delta t_{av}}$	$\frac{\Delta F+F_1}{A}/\Delta t_{av}$	G	G		F	FG								
4	R	I	P	A	$\frac{\Delta p_n}{\Delta t_{av}}$	$\frac{\Delta(F/F_1)}{\Delta t_{av}}$	FG	F		F	FB	B							
5	R	I	P	I	$\frac{\Delta p_n}{\Delta t_{av}}$	$\frac{\Delta F+F_1}{F_1}/\Delta t_{av}$	FG	G											
6	R	I	P	E	$\frac{\Delta p_n}{\Delta t_{av}}$	$\frac{\Delta F+F_1}{\eta F_1}/\Delta t_{av}$				G	FG								Correlations obtained do not justify the extra complexity involved in considering efficiency effects.
7	R	I	P	E	$\frac{\Delta p_n}{\Delta t_{av}}$	$\frac{\Delta F+(1-\eta)F_1}{\eta F_1}/\Delta t_{av}$				F	FB								
8	R	I	P	E	$\frac{\Delta p_n}{\Delta t_{av}}$	$\frac{\Delta F+(1-\eta)F_1}{A \ \Delta t_{av}}$				F	F								
9	RM	I	P	A	$\frac{\Delta p_n}{\Delta t_{av}}$	$\Delta(F/A)$	B	B											
10	RM	I	P	I	$\frac{\Delta p_n}{\Delta t_{av}}$	$\frac{\Delta F+F_1}{A}$	B	B											Bad correlations. Should not be considered.
11	RM	I	P	A	Δp_n	$\frac{\Delta(F/A)}{\Delta t_{fuel}}$				B	B								
12	RM	I	P	I	Δp_n	$\frac{\Delta F+F_1}{A}/\Delta t_{av}$	B												
13	M	-	P	A	Δp_n	$\Delta(F/A)$	FG	FG		B	B		FB	B		FB	FI		The simplification involved in neglecting time effects is justified. The final correlations (16) and (17) in terms of Mach number are considered the most useful information.
14	M	-	P	I	Δp_n	$\frac{\Delta F+F_1}{A}$	FG	FG		B	B		FB	F		F	FI		
15	M	-	P	A	Δp_n	$\Delta(F/F_1)$				VB									
16	M	-	P	A	$\frac{1+M_0}{M_0}\Delta p_n$	$\Delta(F/A)$	FG	FG	FB										
17	M	-	P	I	$\frac{1+M_0}{M_0}\Delta p_n$	$\frac{\Delta F+F_1}{A}$	FG	F	B										

Results quoted are of correlations for all Mach numbers.

4.2   Comparison of theory and experiment

The mean experimental results obtained from Figs. 5 to 7 are compared with the various theoretical calculations in Figs. 8 and 9 for upstream and downstream pulses respectively.

Both figures show the following theoretical results, plotted on the basis of both the fuel increment $\frac{\Delta F}{A}$ and the total fuel flow $\frac{\Delta F + F_1}{A}$:-

1. Small pulse calculation

   (a)   Zero initial fuel flow, any Mach number.

   (b)   Large initial fuel flow, $M_0 = 0.10$ and 0.20

2. Mathematical large pulse calculation

   (a)   Neglecting all attenuations and reflections.

   (b)   Considering the primary attenuation only.

3.   Complete graphical calculation for large pulse and large initial fuel flow, all attenuations and reflections being considered.

The results of these theoretical treatments, all of which assume a simple law of burning, may be compared by means of Figs. 8 and 9 and the following conclusions may be drawn:-

(1)   For small pulses the parameter $\frac{1 \pm M_0}{M_0} \frac{\Delta P}{P}$ is decreased by an increase in initial fuel flow but is far less sensitive to initial Mach number.   It would be expected that the same trends will apply to large pulses.

(2)   Large pulse theory gives a curve of pressure transient against fuel transient, the slope of which at the origin is predicted by the small pulse theory.   Primary attenuation causes relatively more loss to the upstream pulse than to the downstream pulse and since attenuation of one of these will appear as reflection on to the other, it may be expected that, were all reflections considered, the resultant upstream pulse would be slightly below the simple upstream pulse with all reflections and attenuations neglected, whilst the resultant downstream pulse would be above the corresponding simple result.

(3)   The complete graphical calculation for one fuel transient is in good agreement with the mathematical large pulse theory, particularly when the considerations of (2) above are taken into account.   In comparing these in Fig. 9 it should be remembered that results of the large pulse theory as given apply only in the cool region.

Considering now both theoretical and experimental results, the following remarks may be made:-

(1)   The downstream experimental results show far more scatter than the upstream ones and this can probably be explained by the complex effects of initial fuel flow and Mach number and transient size in determining whether or not the downstream pulse is measured in the hottest or coolest region, the theory of section 2.4 showing this to have a large effect on the pulse size.

(2)   The upstream experimental results show consistently lower pressure pulses for pick up 1 than for pick up 2, which may be due to the effect of friction on the 60 in. of duct separating them.

(3)   Experimental results for the downstream pulse agree well with the large pulse theory up to values of $\frac{\Delta P}{A}$ of about 3.0×10^{-3} after which they become too low. This may be because the calculations made from large pulse theory were on the basis of a simple burning law.

(4)   Experimental results for the upstream pulse are lower than predicted by large pulse theory, even at low values of $\frac{\Delta P}{A}$. This effect, which is in contrast to the downstream agreement, is almost certainly due to friction at the flame stabiliser which was not considered in the quoted theoretical results. The experimental results for pick up 2 (just upstream of the stabiliser) are about 30 per cent below large pulse theory at low values of $\Delta P/_A$ and this amount of frictional attenuation has been shown theoretically to be possible using the present stabiliser. (Ref. 4.)

(5)   The reasonable agreement between theory and experiment could be fortuitous since the former assumed 100 per cent combustion efficiency whereas experimental steady state efficiencies before production of the transient are known to have been as low as 50 per cent. However, there is every likelihood that efficiency rises during the transient and it is considered that the assumption of 100 per cent efficiency during the transient is justified.

4.3   Discussion

The large experimental error and the lack of reproducibility of the transient process have made it impossible to obtain very accurate data or to determine experimentally the exact effect of the operating variables. Of the various correlations attempted, those of Figs. 5 to 7 are considered the best on the grounds of both accuracy and simplicity. In the range of initial fuel flow and fuel transient tested (up to 18 per cent and 43 per cent stoichiometric respectively) there is little to choose between correlations on the basis of $\frac{\Delta P}{A}$ or $\frac{\Delta P + F_1}{A}$ but the former is more convenient to use, particularly as it applies for both compression and rarefaction transients.

Considering the various theoretical results (Figs 8 and 9), the small pulse calculation gives the initial slope of the pressure transient-fuel transient curve and indicates the effects of initial fuel flow and Mach number, the former having a far greater effect than the latter on the parameter $\frac{1 + M_0}{M_0} \frac{\Delta p}{p}$. The simple large pulse calculation is in good agreement with the complete numerical treatment when attenuations and reflections are considered.

Comparing theoretical and experimental results (Figs. 8 and 9), these are in good agreement for the downstream pulse up to values of $\frac{\Delta P}{A}$ of about 3.0×10^{-3}, after which the theoretical results are too high, possibly because of assumption of a simple burning law. The upstream results for pick up 2, situated just upstream of the stabiliser, are about 30 per cent below theoretical results, even for small values of $\frac{\Delta P}{A}$ and this is attributed to losses at the flame stabiliser. Experimental

results for pick up 1 are about 23 per cent below those for pick up 2 probably because of friction in the length of duct connecting them.

As regards the application of the present test results to other combustion systems and other fuels, there is no doubt that the different turbulence and combustion rate in these circumstances will lead to different results. However, the reasonable agreement between theoretical calculations and experiment, despite the simple assumptions made, indicate that the results are relatively insensitive to changes in combustion behaviour and hence to changes in rig configuration. The chief item which is likely to change from rig to rig is the loss at the stabiliser and this can be dealt with by the method of Ref. 5.

It should be remembered that the experimental results apply only for an initial fuel flow less than stoichiometric and for pulse sizes which keep the total fuel flow below stoichiometric. In other circumstances the effect could be calculated from the present results on the assumption that no dissociation or variation of combustion rate occurs near stoichiometric. However, this would be only a first approximation and a fuller investigation would be required to find the effect of fuel transients near or above stoichiometric. It should not necessarily be assumed, because a fuel pulse above stoichiometric causes no heat addition, that no pressure pulse will arise, because the effect of this additional fuel in altering the combustion rate can give rise to a pulse.

5.0   Conclusions

The present work, together with the theoretical treatment of References 3, 4, 6 and 8 constitutes an investigation of the production of pressure pulses by a change in fuel-air ratio in a combustion chamber of the ram jet or reheat type.

The theoretical treatments form a basis for understanding the problem and the present experimental investigation may be regarded both as a check of the theoretical results and an experimental determination of data to be used in dealing with this type of problem - for instance in the prediction of stability limits and amplitudes of oscillations of the type dealt with in References 1 and 2.

Calculations have been made based on the theoretical treatments of References 3, 4 and 8 and these have been found to be in good agreement with one another. The calculations show the trend of the effects of initial fuel flow and Mach number and pulse size.

The experimental results show considerable scatter but, nevertheless, reasonable correlations have been obtained. If numerical data of the transient effect is required, it is suggested that the experimental results in terms of $\frac{\Delta P}{A}$ be used, pick up 3 giving the downstream pulse (Fig. 7) and pick up 2 the upstream (Fig. 6), bearing in mind that the latter includes a loss of about 30 per cent due to friction at the stabiliser.

As shown in Figs. 8 and 9 theory is in good agreement with experiment for transients up to $\frac{\Delta P}{A} = 3.0 \times 10^{-3}$ provided the effect of stabiliser friction is considered. For larger transients divergence occurs, probably because of simple assumptions made in the theoretical calculations.

The agreement between experiment and theory is such as to give confidence that the present results can be applied to other rigs and fuels.

LIST OF SYMBOLS

a	sonic velocity
A	air mass flow
C_p	specific heat at constant pressure
F	fuel mass flow (F_1 = initial steady fuel flow)
H	calorific value
k, K	constants
ℓ	length of duct along which fuel transient exists
M	Mach number (M_0 = initial cold Mach number
	M_1 = Mach number at point of heat addition)
p	duct pressure
Q	fuel energy
R	gas constant
S	entropy (suffices o and 1 as for M)
T	absolute static temperature
t	time
v	reflection factor
w	transmission factor (w_u upstream
	(w_d downstream
x	distance along duct
γ	specific heat ratio
Δ	increment (small or finite)
η	combustion efficiency
τ	reaction time

REFERENCES

No.	Author(s)	Title etc.
1.	H. N. Nicholson and ... Radcliffe	Investigation of pressure fluctuations on a Derwent V jet engine. NGTE Report R. 85. Feb. 1951.
2.	D. C. Stewart and K. V. Nesbitt	An Investigation of oscillations of the type found in ramjet and reheat combustion chambers. NGTE Report R. 134. 1953.
3.	P. W. H. Howe	Outlined general treatment of the calculation of wave effects due to small disturbances of steady stabilised burning. Part I. NGTE Report R. 128. Nov. 1952.
4.	D. C. Stewart	The calculation of non-isentropic wave action in a duct, the effect of a fuel-air ratio transient. NGTE Memorandum M. 146 April 1952.
5.	P. W. H. Howe	Reflection of a small pressure pulse by distributed friction in one-dimensional gas flow. NGTE Memorandum M. 168. Feb. 1953 Also unpublished extensions of this work.
6.	P. W. H. Howe	Outlined general treatment of the calculation of wave effects due to small disturbances of steady stabilised burning. Part II. NGTE Report R. 135. 1953.
7.		Handbook of supersonic aerodynamics. Applied Physics Laboratory. The Johns Hopkins University.
8.	P. W. H. Howe	Unpublished work at NGTE on the extension of small pulse theory to that for a large pulse.

Advance Circulation by N·G·T·E·

CS(A)
The Chief Scientist
COWL
DGTD(A)
PDSR(A)
PDRRD
DERD
NA/DERD
DIGT
AD/Eng. R.
AD/Eng. RD.6
DOWRD
TPA3/TIB (M) Dist. 233 copies

APPENDIX I

Approximate Combustion Rate in the Steady State

In the theoretical calculations of Section 2 and Appendix II the assumption is made that before, during and after the transient, combustion is such that the time rate of entropy increase is constant for each layer of gas in the combustion zone.

This assumption is justified on the grounds of the resultant simplification of the calculations, but it is important to determine how accurate the assumption is. It would require very elaborate instrumentation to determine the combustion behaviour during or immediately after the transient but the steady state combustion data before the transient can be obtained and this has been done approximately.

The combustion zone of the duct was fitted with bosses every 6 in. through which a total temperature thermocouple could be inserted and seven runs were made in which the centre point temperature was measured at each station. The runs covered a range of cold Mach numbers from 0.15 to 0.25 and fuel flows corresponding to maximum temperatures from 190°C. to 520°C., attainment of higher temperatures being prevented by the tendency of the rig to oscillate at high fuel flows.

Despite the range of Mach numbers and fuel flows covered, all longitudinal temperature profiles were very similar and the mean experimental results for all seven runs are shown in Fig. 10A. The mean line so obtained indicates the approximate temperature variation but this should not be taken too literally as only centre point temperatures were measured and insufficient points were taken to indicate the true variation, particularly near the origin where a straight or rounded toe may be assumed.

Assuming the mean total temperature variation of Fig. 10A with the mean initial Mach number and air temperature, the tables of Ref. 7 have been used to find the corresponding Mach number and sonic velocity variation whence the time corresponding to each duct position was found by graphical integration of the expression:-

$$t = \int \frac{dx}{Ma}$$

The entropy variation was also found by graphical integration using the expression:-

$$\frac{S}{C_p} = \int \left(1 + \frac{\gamma - 1}{2} M^2\right) \frac{dT_0}{T_0} \quad \text{(from Ref. 6)}$$

$$(T_0 = \text{Total temperature})$$

The results of this are shown in Fig. 10B as graphs of total temperature and entropy against time. It will be noted that the assumption of constant rate of entropy increase is less accurate than would be one of linear increase in total temperature.

Note that these results form merely an approximate guide to the combustion behaviour and should not be interpreted as an accurate determination of combustion data.

APPENDIX II

Detailed Theoretical Calculations

1.   Small pulse calculation

For zero initial fuel flow, equation (1), Section 2.2 (i.e. equations (32) and (34) of Ref. 2) is used directly:-

$$\frac{1 \pm M_0}{M_0} \frac{\Delta p}{p} = \frac{\gamma}{2 \, C_p} \qquad \Delta S \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

Assuming an initial temperature of $40°C$., $\gamma = 1.4$ and $C_p = 0.24$ and taking the fuel as hydrogen having a calorific value of $H = 28650$ CHU./lb., leads to:-

$$\Delta S = \frac{\Delta Q}{T} = \frac{H}{T} \qquad \Delta(F/_A) \text{ with the obvious notation}$$

whence

$$\frac{1 \pm M_0}{M} \frac{\Delta p}{p} = \frac{\gamma}{2 \, C_p} \frac{H}{T} \qquad \Delta(F/_A)$$

$$= \frac{1.4}{2 \times 0.24} \frac{28.650}{313} \quad \Delta(F/_A)$$

$$= 0.267 \quad \Delta(F/_A) \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(2)$$

where $F/_A$ is now expressed as milli pounds per pound.

For a large initial fuel flow, the upstream pulse is attenuated in passing through the burning zone and the portion of it reflected is added to the downstream pulse. The proportions attenuated and reflected are given by equations (64) and (68) of Ref. 3 as:-

Proportion remaining after attenuation:

$$w = e^{-\frac{1}{4} \int_{S_0}^{S_1} \frac{1 + \gamma M^2}{(1-M)^2} \frac{dS}{C_p}} \qquad \dots\dots\dots\dots\dots(4)$$

Proportion reflected:

$$v = - \int_{1}^{w_0} \left(\frac{1 - M}{1 + M}\right)^2 \, dw \qquad \dots\dots\dots\dots\dots(5)$$

Assuming 20 per cent stoichiometric fuel flow (stoichiometric $F/_A = 28.9 \times 10^{-3}$ for hydrogen-air), the temperature rise across the burning zone is

$$\Delta T = \frac{F/A \, H}{C_p} = \frac{0.20 \times 28.9 \times 10^{-3} \times 28650}{0.25}$$

$$= 662° \text{ C.} \qquad \text{(assuming } C_p = 0.25 \text{ at this higher temperature)}$$

So final temperature $= 313 + 662 = 975°K$.

Thus the pulses initially produced at this temperature are given by the equation:-

$$\frac{1 \pm M_1}{M_1} \frac{\Delta p}{p} = \frac{1.4}{2 \times 0.25} \frac{28.650}{975} \quad \Delta(F/A)$$

$$= 0.0823 \; \Delta(F/A) \; \dots\dots\dots\dots\dots\dots\dots\dots\dots(6)$$

where M_1 now refers to the Mach number at the higher temperature.

The attenuations and reflections of the basic upstream pulse depend on the Mach number variation through the burning zone. Using the tables of Ref. 7 for steady flow with heat addition, the entropy and Mach number variation through the burning zone has been calculated for an inlet temperature of 313°K. and outlet temperature of 975°K. and for initial Mach numbers of 0.10 and 0.20. Using this information, w and v have been evaluated from equations (4) and (5). Results are as follows:-

	0.10	0.20
Initial cold Mach number M_0	0.10	0.20
Initial total temperature	313°K.	313°K.
Final total temperature	975°K.	975°K.
Final Mach number M_1	0.182	0.408
Entropy increase through burning zone $\frac{\Delta S}{C_p}$	1.143	1.143
Transmission, w from equation (4)	0.672	0.524
Reflection, v from equation (5)	0.189	0.135

The resultant values of upstream and downstream pulse considering attenuations and reflections and expressed with a factor in terms of initial Mach number for comparison with equation (2) are:-

Upstream pulse:

$$\frac{1 - M_0}{M_0} \frac{\Delta p}{p} = \frac{1 - M_0}{M_0} \left\{ \frac{M_1}{1-M_1} \quad w \times 0.0823 \quad \Delta(F/A) \right\}$$

using equation (6) for the basic pulse.

Substituting for M_0, M_1 and w, this becomes:-

$$\frac{1 - M_0}{M_0} \frac{\Delta p}{p} = 0.111 \quad \Delta(F/A) \text{ for } M_0 = 0.1$$
$$= 0.119 \quad \Delta(F/A) \text{ for } M_0 = 0.2 \Big\} \quad \dots\dots\dots\dots(7)$$

Downstream pulse

$$\frac{1 + M_0}{M_0} \frac{\Delta p}{p} = \frac{1 + M_0}{M_0} \left\{ \frac{M_1}{1 + M_1} \quad \frac{v \; M_1}{1-M_1} \right\} \times 0.0823 \; \Delta(F/A)$$

which similarly becomes

$$\frac{1 + M_0}{M_0} \frac{\Delta p}{p} = 0.178 \quad \Delta(F/A) \text{ for } M_0 = 0.1$$
$$= 0.189 \quad \Delta(F/A) \text{ for } M_0 = 0.2 \Big\} \quad \dots\dots\dots\dots(8)$$

2.　Mathematical large pulse calculation

The extension from small pulse to large pulse theory has been carried out in Ref. 8 and may be described with reference to Fig. 11 which represents events on the Lagrange plane. This type of diagram is fully explained in Refs. 3 and 4 and it is sufficient here to say that the ordinate and abscissa of such a diagram represent respectively time and the Lagrange co-ordinate which is defined as the mass of gas contained between a particular gas layer and a datum layer.

In Fig. 11 AB represents the flame stabilisers and CD and EF the initial and final layers of gas in which the large fuel transient exists. Combustion starts at the flame stabiliser CE and finishes along the line CF.

Considering a small combustion increment GH, upstream and downstream pressure pulses are propagated along characteristics and may be measured in the isentropic regions at JK and LM respectively.

Small pulse theory gives the pressure pulses of the combustion pulse GH measured along the isentropic lines GN and GP, these lines being, respectively, vertical and parallel to AB.

It is shown in Ref. 8 that the finite sum of small pulses such as GP and GN can be found by integration of small pulse theory along the line CF. The result expresses the upstream and downstream pulses in terms of the heat addition ΔQ implicitly using the Mach number M along CF as parameter thus:-

Upstream pulse:

$$\Delta a = -\frac{\gamma-1}{\gamma+1} \frac{\sqrt{\gamma R}}{K} \int_{M_0}^{M} \frac{1+M}{1-M} \frac{1}{M^2} e^{\frac{2}{(\gamma+1) M}} dM \quad \ldots\ldots(9)$$

Downstream pulse:

$$\Delta a = -\frac{\gamma-1}{2} \frac{\sqrt{\gamma R}}{K} \left[e^{\frac{2}{(\gamma+1) M}} \right]_{M_0}^{M} \quad \ldots\ldots\ldots\ldots\ldots(10)$$

Heat addition:

$$\Delta Q = -\frac{4 C_p}{K^2 (\gamma+1)} \int_{M_0}^{M} \frac{(1+M)}{M^4} e^{\frac{4}{(\gamma+1) M}} dM \quad \ldots\ldots\ldots(11)$$

where a represents sonic velocity and the constant K is obtained from the initial conditions:-

$$K = \frac{e^{\frac{2}{(\gamma+1) M_0}}}{M_0 T_0^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Rearranging these to express pulse size in terms of pressure change Δp and substituting for the following data:

Initial Temperature	T_0	$= 313^\circ K.$
" Mach number	M_0	$= 0.20$
Specific heat	C_p	$= 0.24$
" " ratio	γ	$= 1.4$
Calorific value	H	$= 28650$ CHU./lb.

the equations (9) to (12) become:

upstream pulse

$$\frac{\Delta p}{P_0} = \left\{ 1 + 5.165 \times 10^{-4} \left[-I_2 \right]_{M_0}^{M} \right\}^7 - 1 \quad\ldots\ldots\ldots(13)$$

downstream pulse:

$$\frac{\Delta p}{P_0} = \left\{ 1 + 6.20 \times 10^{-4} \left[I_3 \right]_{M_0}^{M} \right\}^7 - 1 \quad\ldots\ldots\ldots(14)$$

heat addition:

$$\frac{\Delta F}{A} = 2.521 \times 10^{-5} \left[-\frac{I_1}{0.6} \right] \quad\ldots\ldots\ldots(15)$$

(with $\Delta F/A$ in milli lb. per lb.)

where

$$I_1 = \int \frac{1+M}{M^4} e^{\frac{4}{(\gamma+1) M}} dM$$

$$I_2 = \int \frac{1+M}{1-M} \frac{1}{M^2} e^{\frac{2}{(\gamma+1) M}} dM$$

$$I_3 = e^{\frac{2}{(\gamma+1) M}}$$

In Ref. 8 the integrals are evaluated for a range of values of M and the quantities of equations (13) to (15) have thus been calculated as follows:-

M	$-\left[\frac{I_1}{0.6}\right]_{0.2}^{M}$	$\left[-I_2\right]_{0.2}^{M}$	$\left[I_3\right]_{0.2}^{M}$	Upstream $\frac{\Delta p}{P}$	Downstream $\frac{\Delta p}{P}$	$\frac{\Delta F}{A} \times 10^3$
0.20	0	0	0	0	0	0
0.19	0.723×10^5	28.20	15.81	0.109	0.070	1.822
0.18	2.133×10^5	66.80	37.96	0.266	0.179	5.375
0.17	5.070×10^5	121.70	70.06	0.532	0.344	12.78

These results are shown in Figs. 8 and 9 as large pulse theory neglecting attenuation and reflection. They refer to the finite sum of small pulses such as GH and GP.

A closer approximation to actual conditions is to consider attenuation of the upstream and downstream pulses in the burning and burnt zones respectively to the left of line CF (Fig. 11) and to the right of this line. Reflection of these pulses is ignored as the total magnitude of resultant pulses including reflections will depend on the ratio of the combustion rate to the rate of increase of fuel air ratio.

The attenuation factors for burning and burnt regions are derived in Ref. 3 and Ref. 6 respectively. They are only approximate in this case as they apply strictly only for steady conditions before the pulse. They are:-

Burning zone (i.e. upstream pulse):

$$\text{Proportion remaining } w_u = e^{-\frac{1}{4} \int_0^{\Delta S} \frac{1 + \gamma M^2}{(1-M)^2} \frac{dS}{C_p}} \qquad \dots\dots\dots\dots(4)$$

Burnt zone (i.e. downstream pulse)

$$\text{Proportion remaining } w_d = e^{-\frac{\Delta S}{4 \, C_p}} \qquad \dots\dots\dots\dots\dots\dots\dots\dots(16)$$

The upstream attenuation factor was evaluated by first assuming 50 per cent attenuation from which, knowing the initial pulse size, the variation of Mach number at the stabiliser AB was found. The initial and final Mach number along lines such as GJ was thus known and a linear variation of Mach number with entropy was assumed along such lines, the error due to inaccuracy in this assumption being slight. Results were as follows:-

Mach number along CF (Fig. 11)	0.20	0.19	0.18	0.17
First approximation to Mach number along line AB (by assuming 50 per cent attenuation)	0.20	0.162	0.112	0.041
Entropy change along characteristic GJ $\frac{\Delta S}{C_p}$	0	0.592	1.436	2.597
Attenuation factor (equation (4)) w_u	1.00	0.797	0.600	0.492

The downstream attenuation factor was evaluated directly from equation (16) thus:-

Attenuation factor	w_d	1.00	0.862	0.698	0.522

Since the attenuation factors so found apply only to the pulse increment at any point they must be multiplied not by the final large pulse previously found but by successive increments of this pulse. This was achieved by graphically differentiating the large pulse curve previously found, multiplying by the appropriate attenuation factor and graphically integrating. The final results for the attenuation pulses are as follows:-

$\Delta F/A \times 10^3$	Upstream		Downstream	
	$\dfrac{1 - M_0}{M_0}$	$\Delta p/p_0$	$\dfrac{1 + M_0}{M_0}$	$\Delta p/p_0$
0	0		0	
2	0.414		0.448	
4	0.682		0.752	
6	0.880		0.976	
8	1.0468		1.1656	
10	1.1980		1.3341	
12	1.3394		1.4863	

The resultant curves of attenuated pulse size against fuel-air ratio change are shown in Figs. 8 and 9.

3.   Complete graphical calculation

The graphical calculation of Ref. 4 used the following data:

Initial cold Mach number $M_0 = 0.20$

Entropy increase through burner
    before transient $\dfrac{\Delta S}{C_p} = 0.765$

Ratio of entropy increase through
    burner after transient to that
    before transient $\dfrac{\Delta S'}{\Delta S} = 2.0$

                        and $\dfrac{a_0 \tau}{\ell} = 7.36$

where a_0 = initial cold sonic velocity

    τ = duration of combustion reaction before transient

    ℓ = length of fresh mixture through which the "potential"
            entropy increases linearly.

One set of values of the latter three quantities which would be
typical of the present experimental results is:-

        a_0 = 1164 ft./sec. (i.e. initial mixture temperature 40°C.)

        τ = 1.0 milli-sec.

        ℓ = 0.158 ft.

Using the tables of Ref. 7, steady combustion from an initial cold
Mach number 0.20 with an entropy increase $\dfrac{\Delta S}{C_p} = 0.765$ corresponds to an
increase in total temperature of 2.14 times and twice this entropy increase
corresponds to a total temperature increase of 4.45 times.

        Thus the final temperature before the transient is 2.14 x 313 = 669°K.
                    and that after the transient is 4.45 x 313 = 1395°K.

Assuming C_p = 0.240 in the lower temperature range and 0.250 in the higher, the fuel-air ratios to give these temperatures are given by

$$C_p \, \Delta T = H \frac{F}{A}$$

$$\frac{F}{A} = \frac{C_p \, \Delta T}{H} = \frac{0.240 \, (669 - 313)}{28650} = 2.985 \times 10^{-3}$$

for the initial steady state

$$= \frac{0.250 \, (1395 - 313)}{28650} = 9.440 \times 10^{-3}$$

for the final state

Hence the change during combustion is given by

$$\frac{\Delta F}{A} = 6.455 \times 10^{-3}$$

The pulse magnitudes in Ref. 4 are taken from the curves of pressure variation, the increase in pressure up to the peak being divided by the initial steady pressure. The results are:-

For the upstream pulse $\frac{\Delta p}{p} = \frac{1.300 - 1.0}{1.0} = 0.300$

For the downstream pulse

Case 1 (hot region) $\frac{\Delta p}{p} = \frac{1.265 - 0.940}{0.940} = 0.345$

Case 2 (cold region) $\frac{\Delta p}{p} = \frac{1.180 - 0.940}{0.940} = 0.255$

Note:

The data of this problem is completely specified by the parameters:

$$M_o, \quad \frac{\Delta S}{C_p}, \quad \frac{\Delta S'}{\Delta S}, \quad \frac{a_o \tau}{\ell}$$

The curve of pressure variation is plotted against a non dimensional time $\frac{t}{\tau}$ so that the actual time to reach peak pressure is proportional to τ. Also the time during which the fuel increment is introduced is equal to the length of duct along which fuel air ratio varies divided by duct velocity, i.e. this time is proportional to $\frac{\ell}{a_o m_o}$

Hence, given M_o, the parameter $\frac{a_o \tau}{\ell}$ could be replaced by $\frac{a_o M_o \tau}{\ell}$ i.e. by the ratio $\frac{\text{Time to reach peak pressure}}{\text{Time of increase of fuel air ratio}}$ so that the data of the problem could equally well be expressed by the alternative four parameters:
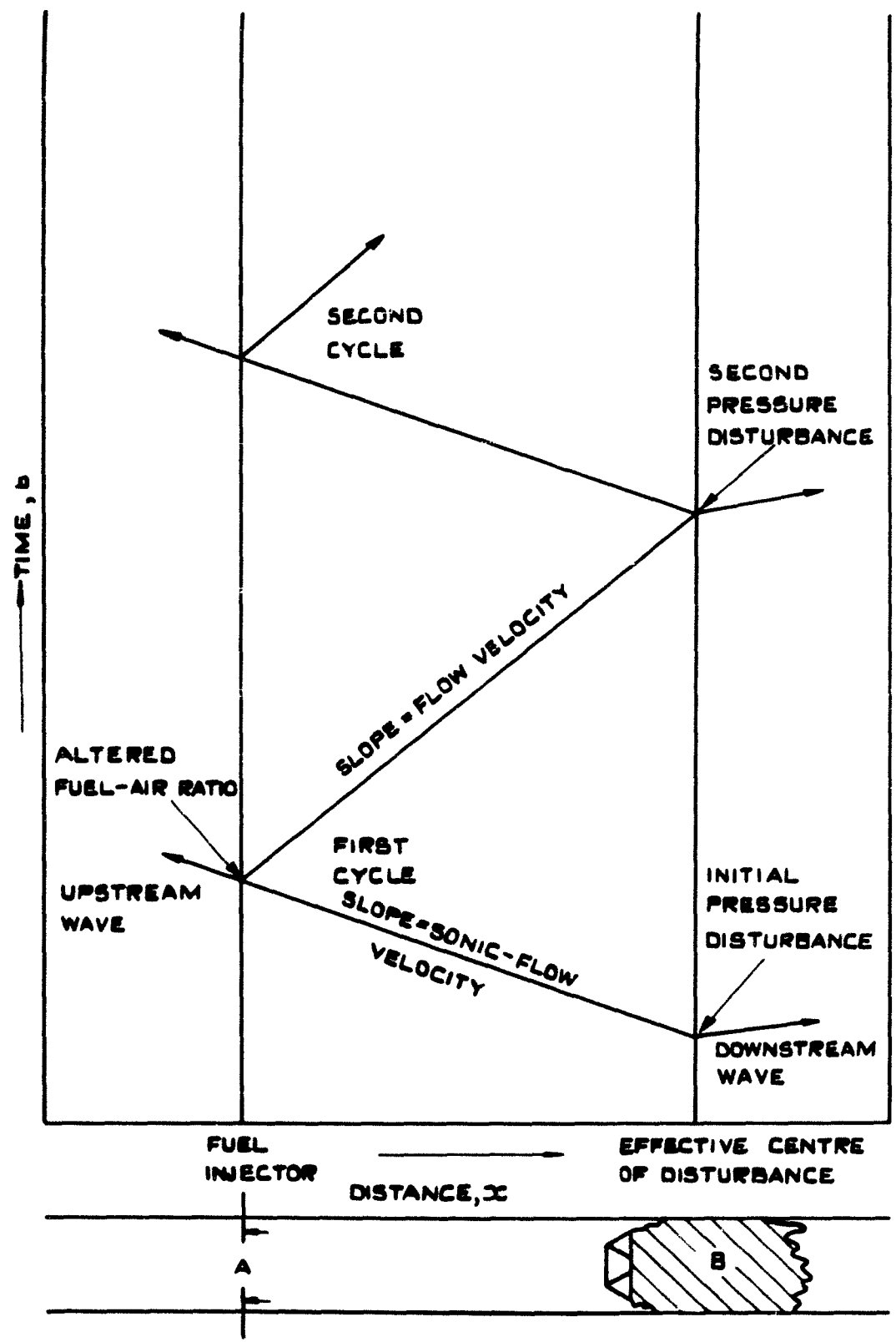
$$M_o, \quad \frac{\Delta S}{C_p}, \quad \frac{\Delta S'}{\Delta S}, \quad \frac{\text{Time to reach peak pressure}}{\text{Time of increase of fuel air ratio}}$$

i.e. by the initial Mach number and fuel flow, the magnitude of the fuel transient and the ratio of the pressure to fuel transient times.
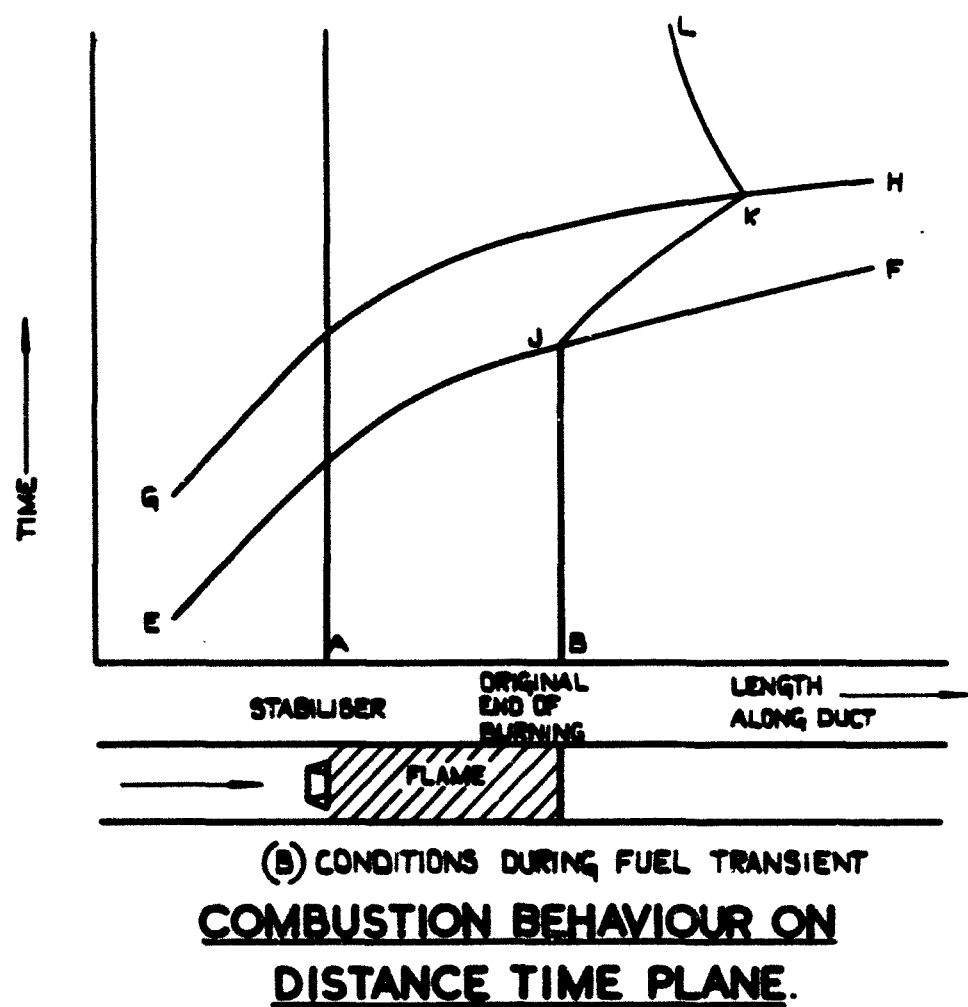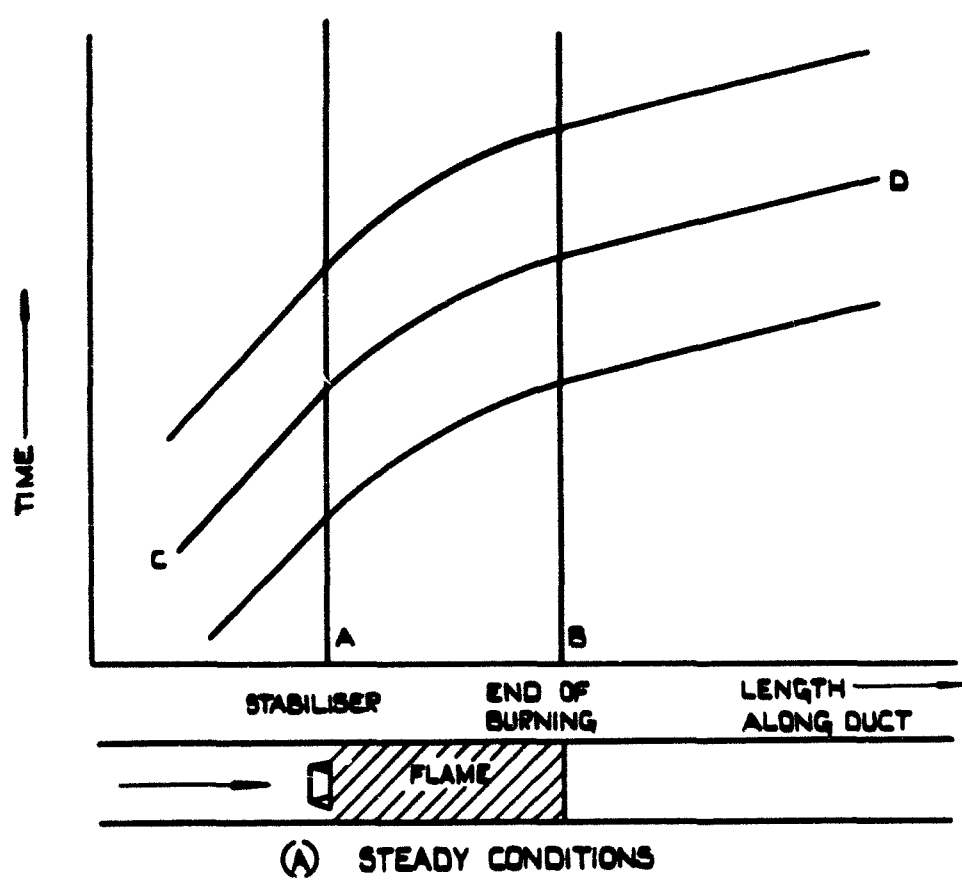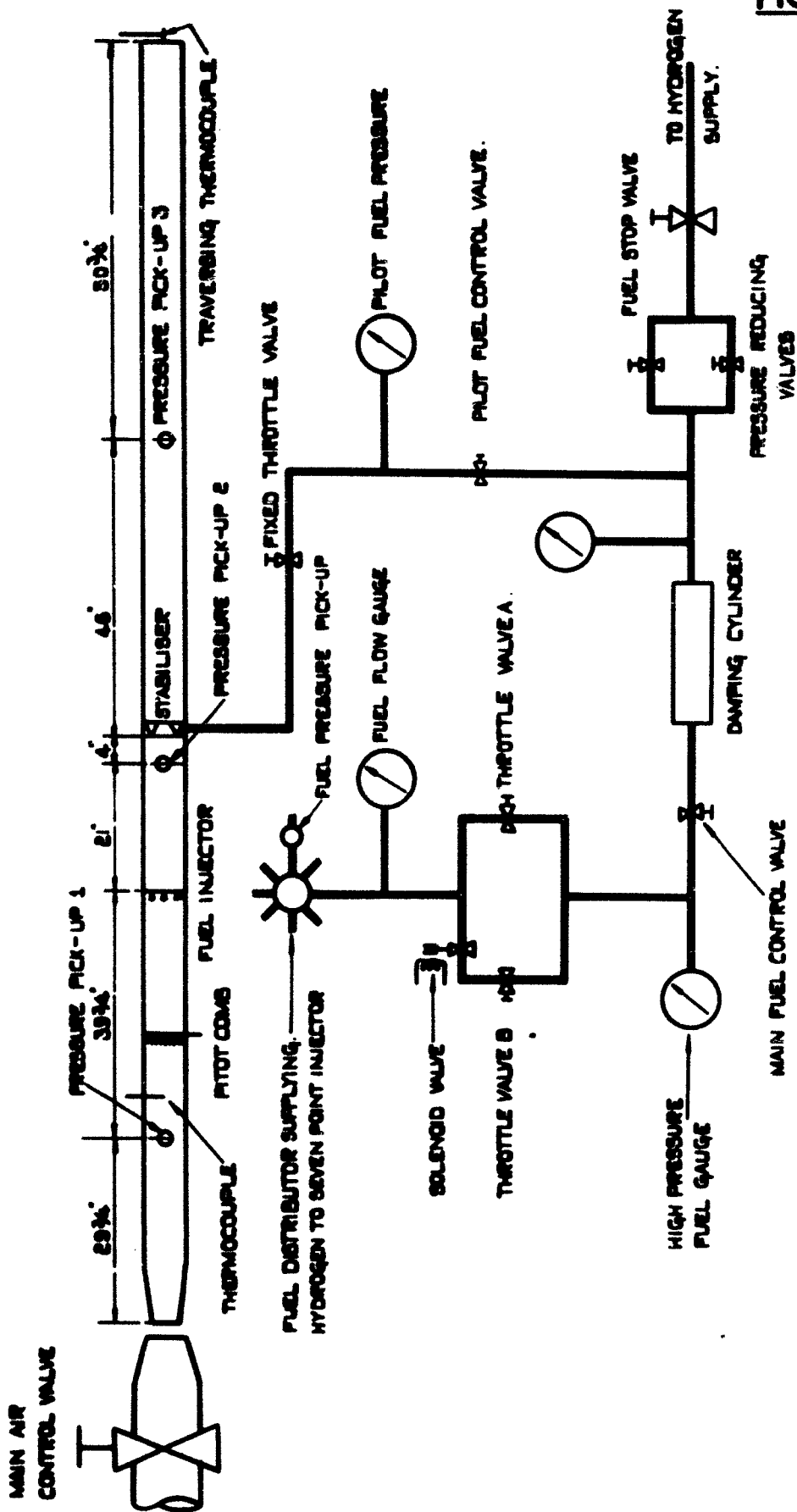
MECHANISM OF OSCILLATION IN A DUCT

FIG. 2.

SK5721i

STABILISER   END OF
BURNING   LENGTH
ALONG DUCT

FLAME

(A)   STEADY CONDITIONS

STABILISER   ORIGINAL
END OF
BURNING   LENGTH
ALONG DUCT

FLAME

(B) CONDITIONS DURING FUEL TRANSIENT

COMBUSTION BEHAVIOUR ON
DISTANCE TIME PLANE.

SK 57212

FIG. 3.

MAIN AIR CONTROL VALVE

THERMOCOUPLE

PRESSURE PICK-UP 1

PITOT COMB

FUEL INJECTOR

FUEL DISTRIBUTOR SUPPLYING HYDROGEN TO SEVEN POINT INJECTOR

PRESSURE PICK-UP 2

STABILISER

FUEL PRESSURE PICK-UP

FIXED THROTTLE VALVE

PRESSURE PICK-UP 3

TRAVERSING THERMOCOUPLE

PILOT FUEL PRESSURE

PILOT FUEL CONTROL VALVE.

FUEL STOP VALVE

TO HYDROGEN SUPPLY.

PRESSURE REDUCING VALVES

FUEL FLOW GAUGE

THROTTLE VALVE A.

SOLENOID VALVE

THROTTLE VALVE B

DAMPING CYLINDER

MAIN FUEL CONTROL VALVE

HIGH PRESSURE FUEL GAUGE

SCHEMATIC DIAGRAM OF 3" RIG INSTALLATION

FIG. 4.

SK. 57213.
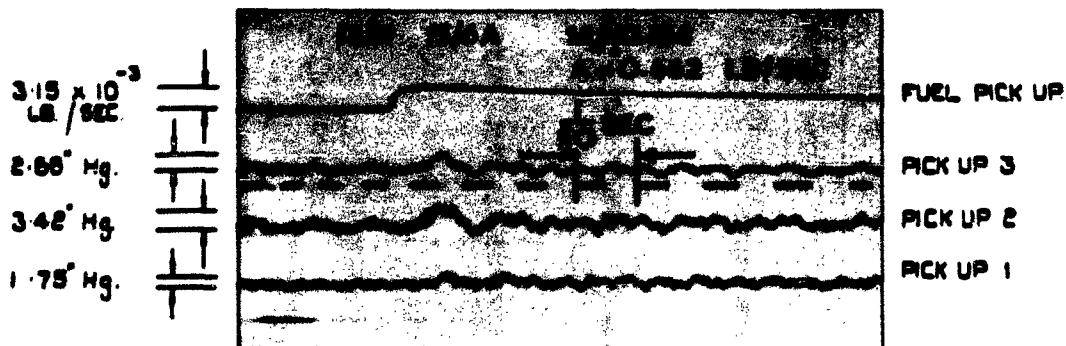
(a) SMALL TRANSIENT
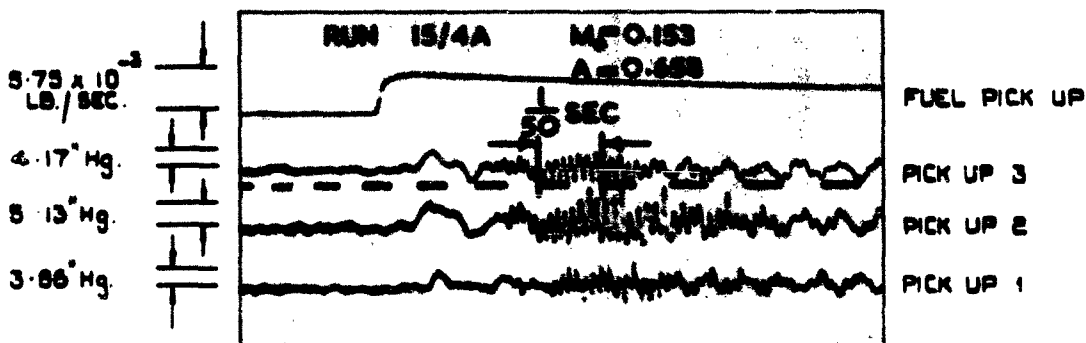
(b) LARGE TRANSIENT
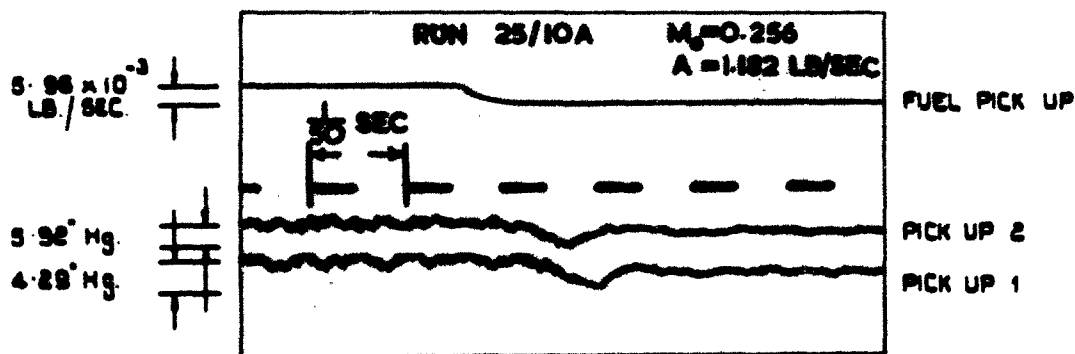
(c) RAREFACTION TRANSIENT

TYPICAL EXPERIMENTAL RECORDS.

FIG.5.

EXPERIMENTAL RESULTS (PICK UP I.)

LEGEND.

COMPRESSION
TRANSIENTS

•	$M_o = 0.15$	(NOMINAL)
×	$M_o = 0.20$	
+	$M_o = 0.25$	
■	$M_o = 0.30$	
●	$M_o = 0.20$	

INJECTOR-STABILISER
LENGTH 25"

STATISTICAL EXPERIMENTS
WITH VARIOUS INJECTOR-STABILISER LENGTHS
AS INDICATED

RINGED POINTS REFER TO RAREFACTION TRANSIENTS

AVERAGE EXPERIMENTAL RESULT

PRESSURE
TRANSIENT
(PICK UP I.)

$\dfrac{1-M_o}{M_o} \dfrac{\Delta P}{P}$

FUEL TRANSIENT $\dfrac{\Delta F}{A}$

AVERAGE EXPERIMENTAL RESULT

PRESSURE
TRANSIENT
(PICK UP I.)

$\dfrac{1-M_o}{M_o} \dfrac{\Delta P}{P}$

FUEL TRANSIENT $\dfrac{\Delta F + F_i}{A}$

FIG.6

57215.
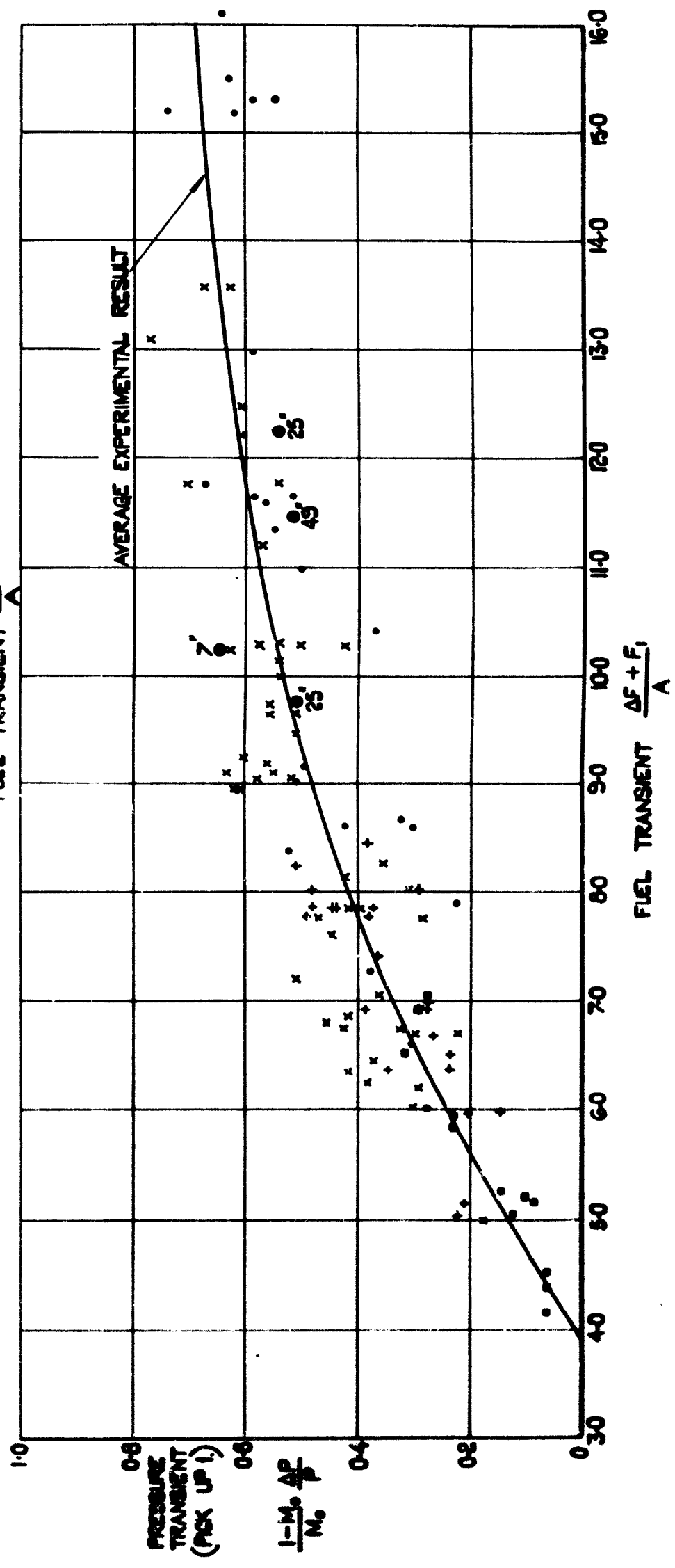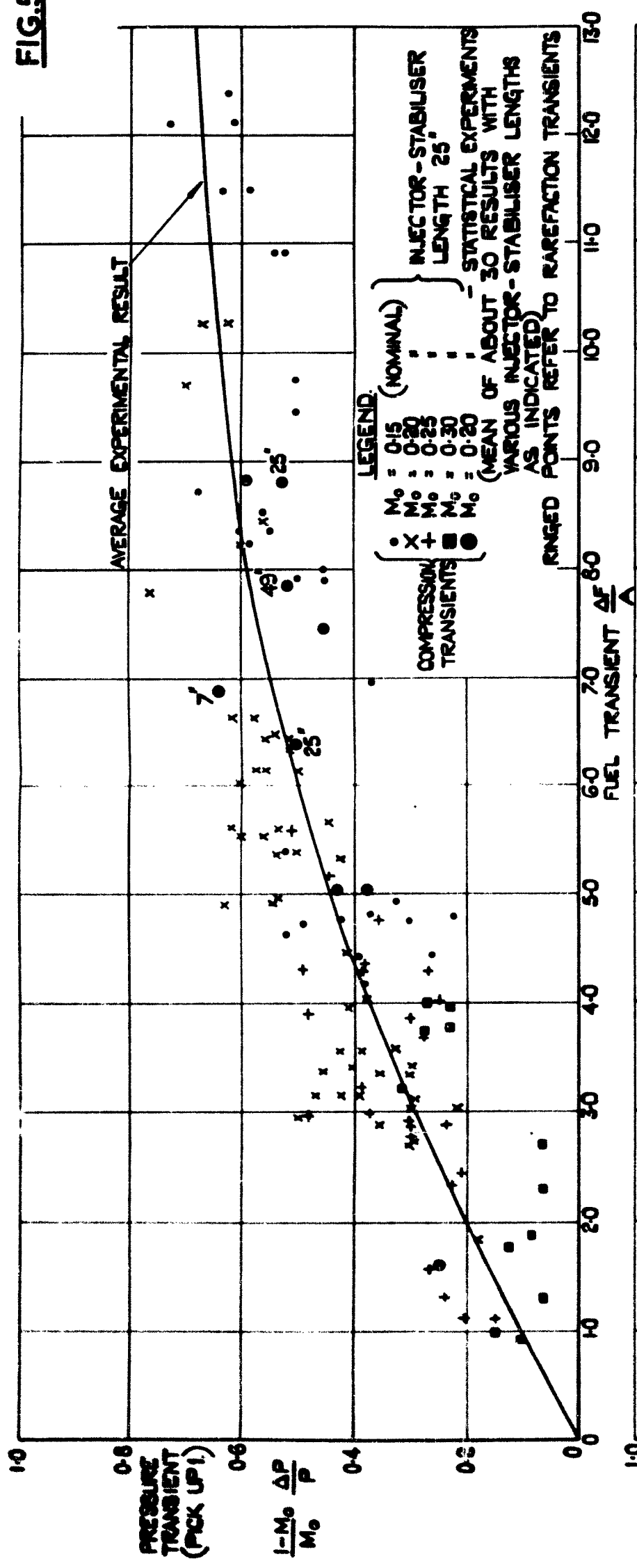
EXPERIMENTAL RESULTS (PICK UP 2.)
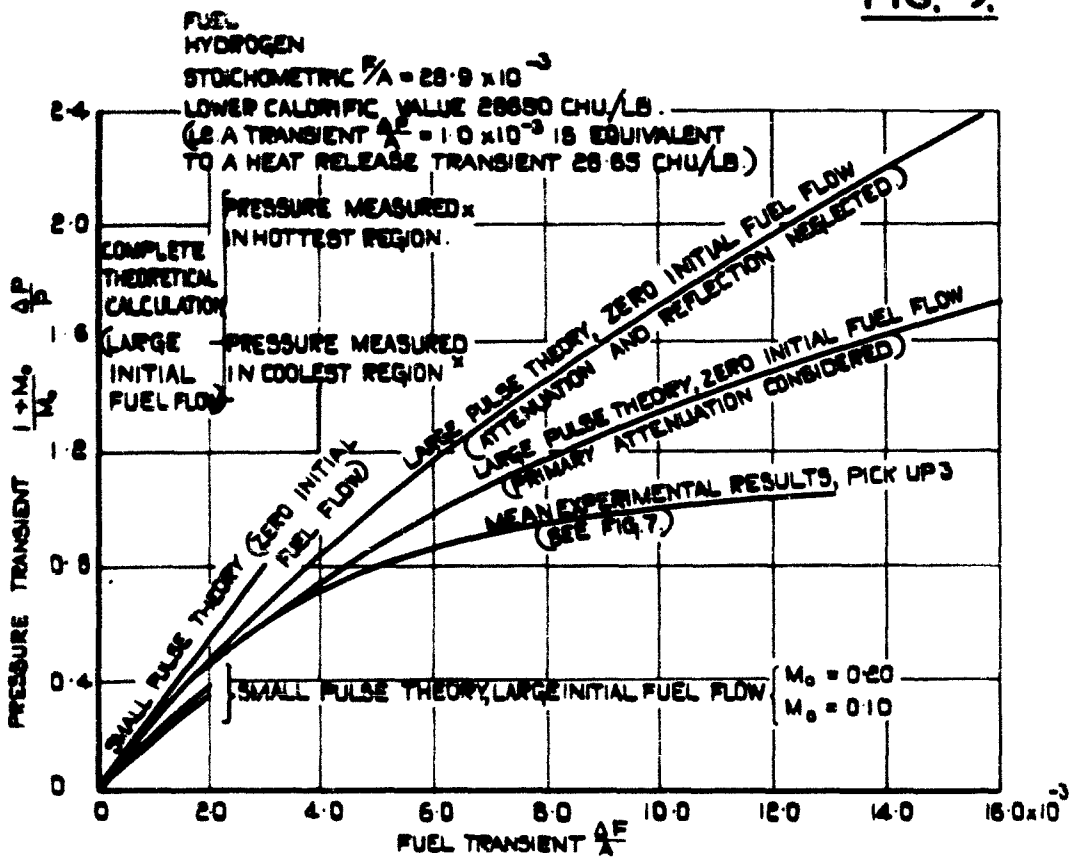
FIG.7.
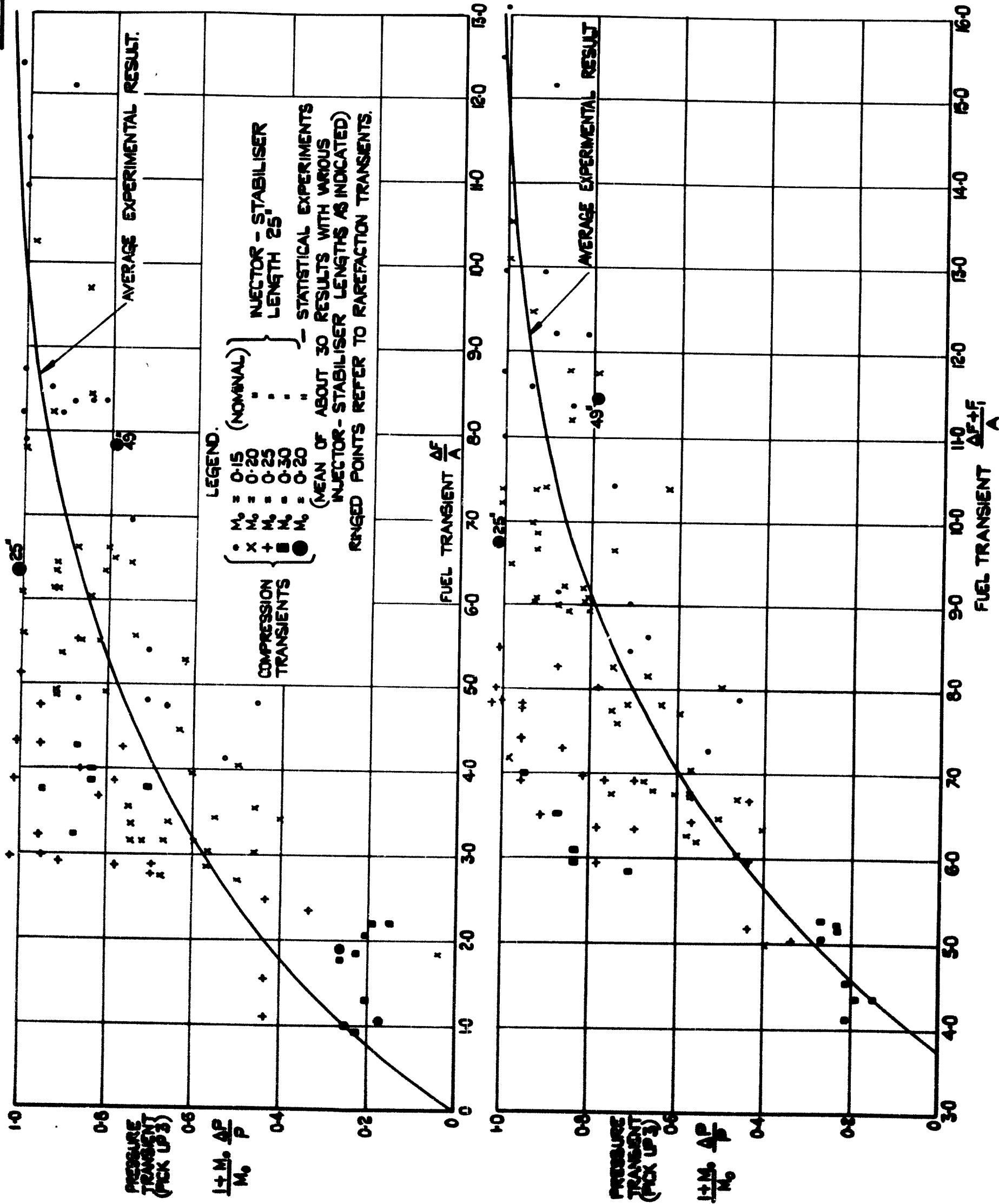
EXPERIMENTAL RESULTS (PICK UP 3.)

57216.
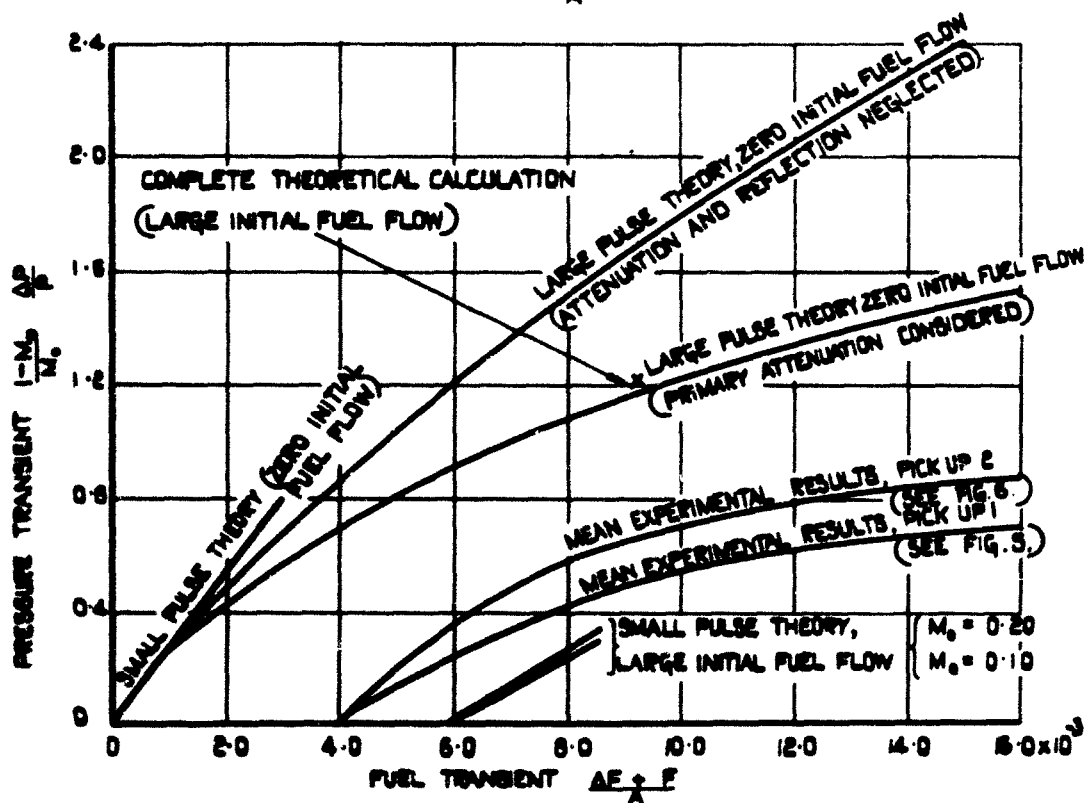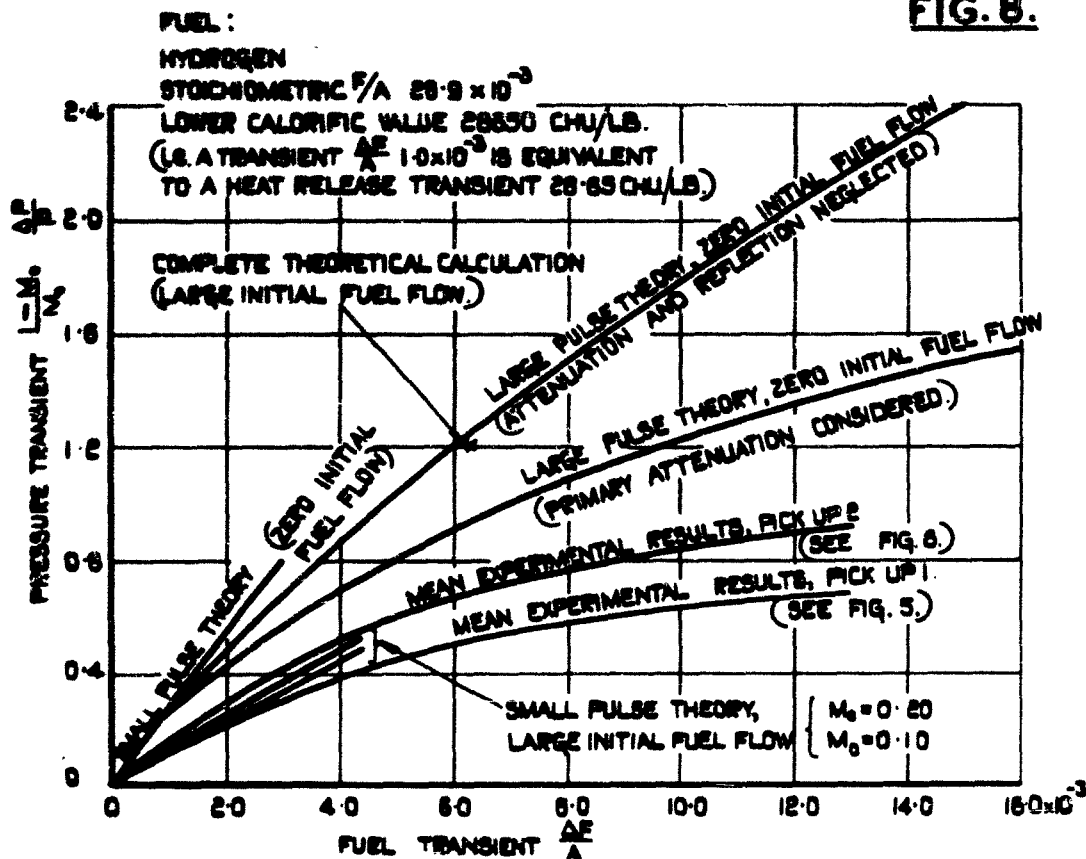
THEORETICAL AND EXPERIMENTAL
RESULTS (UPSTREAM)

FIG. 9.
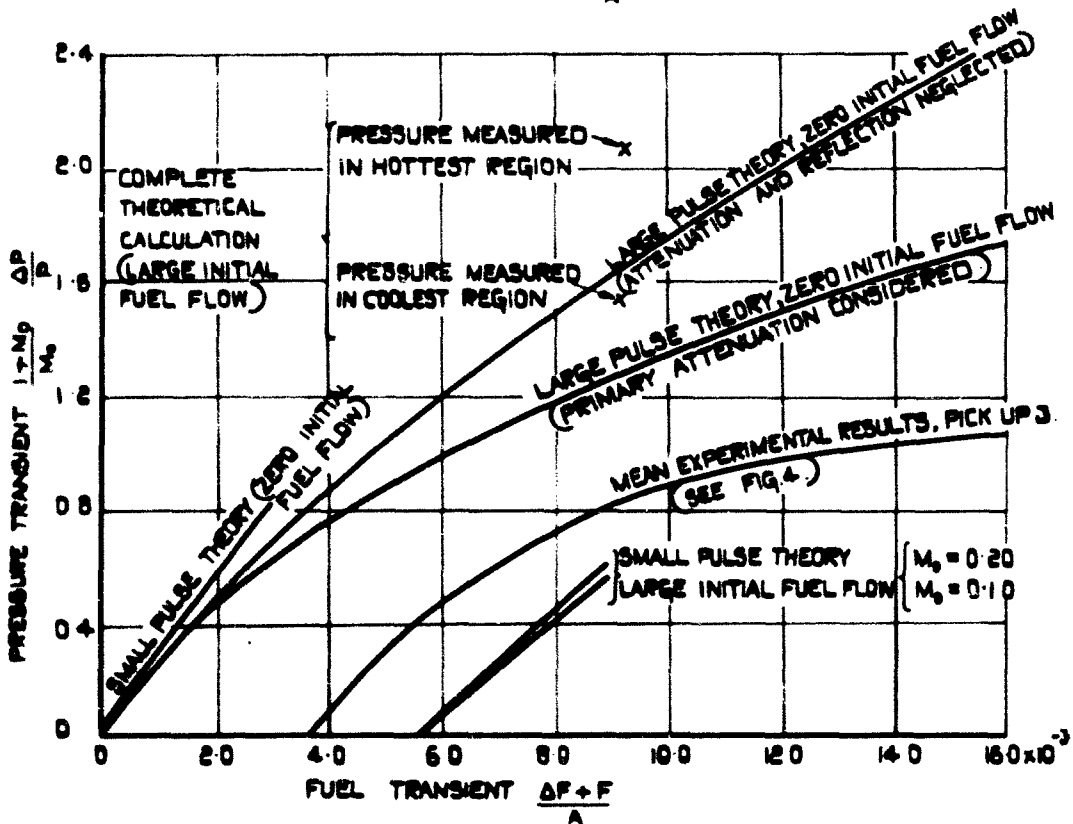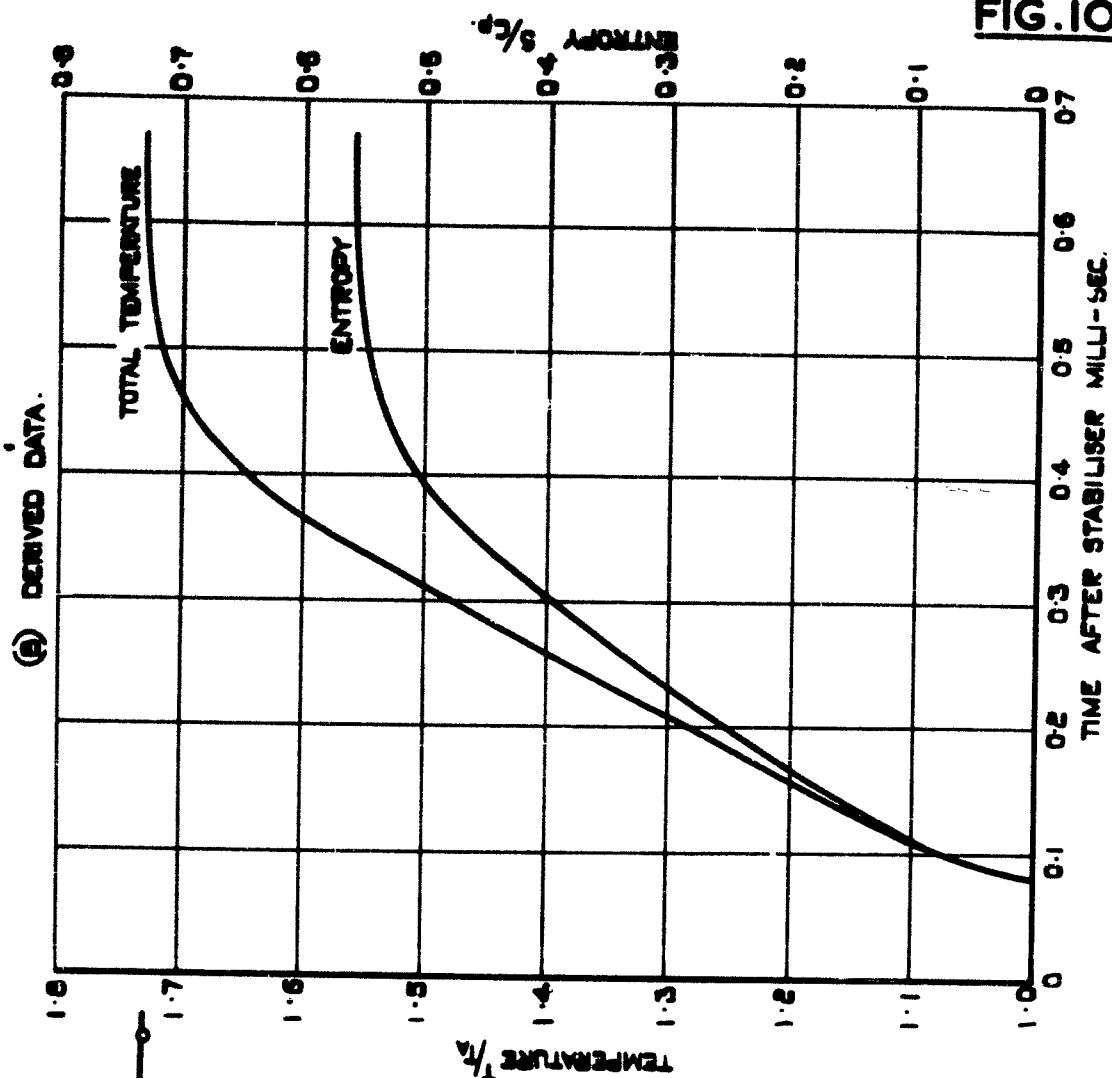
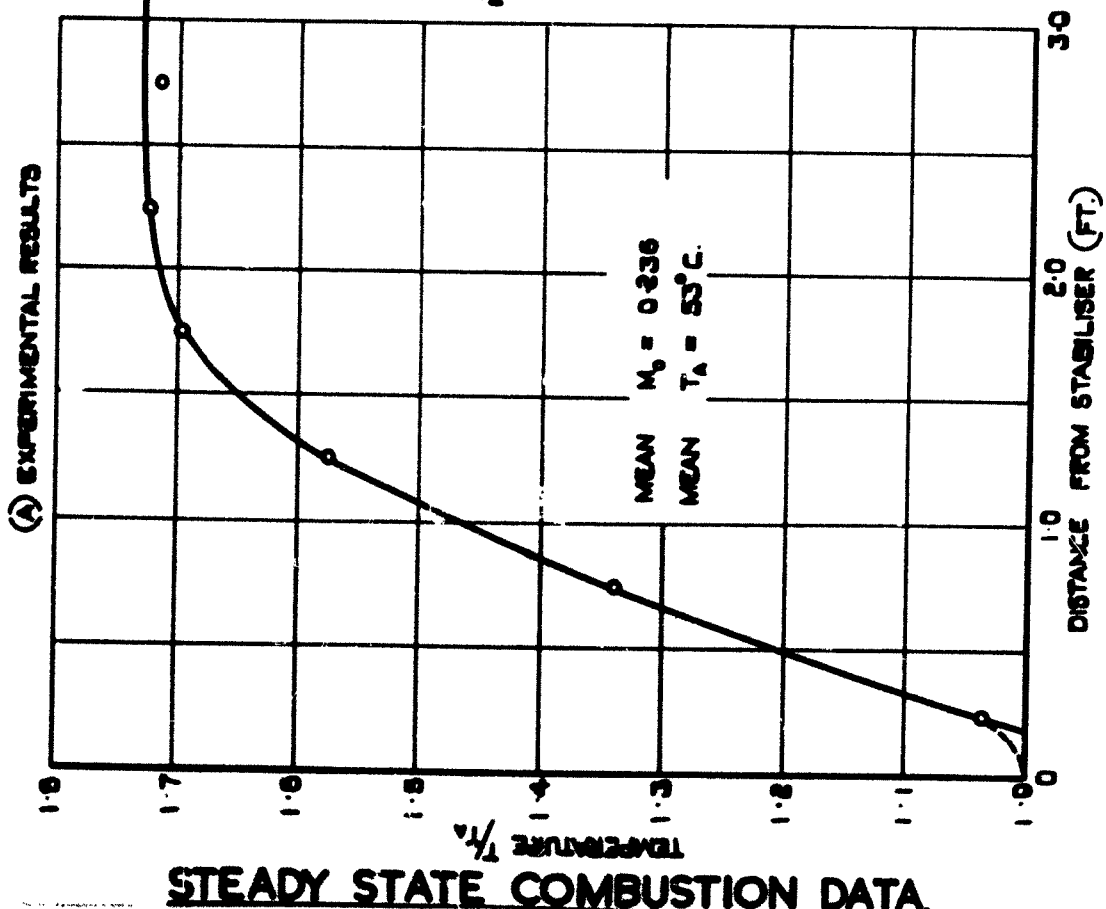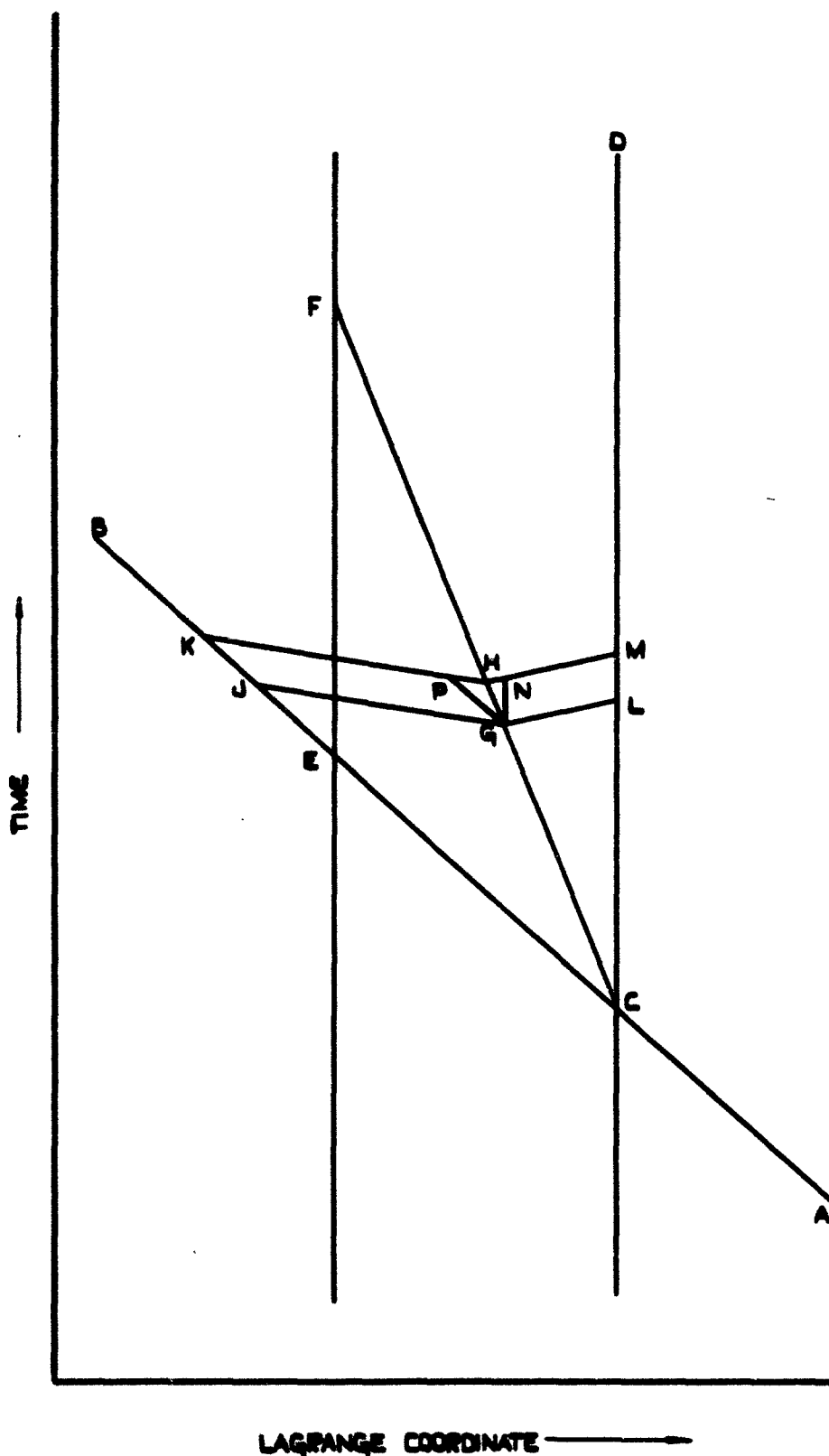THEORETICAL AND EXPERIMENTAL
RESULTS (DOWNSTREAM)

FIG. 10.

SK 57219

STEADY STATE COMBUSTION DATA.

FIG. 11.

SK 57220

LARGE PULSE THEORY—LAGRANGE PLANE.

[dstl]

Information Centre
Knowledge Services
[dstl] Porton Down,
Salisbury
Wilts
SP4 0JQ
22060-6218
Tel: 01980 613753
Fax 01980-613970

Defense Technical Information Center (DTIC)
8725 John J. Kingman Road, Suit 0944
Fort Belvoir, VA   22060-6218
U.S.A.

AD#: AD0011220

Date of Search: 14 May 2009

Record Summary: AVIA 28/3679
    Title: Pressure pulses caused by change in fuel air ratio in a simply stabilised burning zone
    Availability Open Document, Open Description, Normal Closure before FOI Act: 30 years
    Former reference (Department) R132
    Held by The National Archives, Kew

This document is now available at the National Archives, Kew, Surrey, United Kingdom.

DTIC has checked the National Archives Catalogue website (http://www.nationalarchives.gov.uk) and found the document is available and releasable to the public.

Access to UK public records is governed by statute, namely the Public Records Act, 1958, and the Public Records Act, 1967.
The document has been released under the 30 year rule.
(The vast majority of records selected for permanent preservation are made available to the public when they are 30 years old. This is commonly referred to as the 30 year rule and was established by the Public Records Act of 1967).

This document may be treated as <u>UNLIMITED</u>.